Modeling and Mapping of Magnetic Stars

Ewald Gerth

D-14471 Potsdam, Gontardstr 130, Germany.

Yurij V. Glagolevskij

Special Astrophysical Observatory of Russian AS,
Nizhnij Arkhyz 357147, Russia.

Abstract. Models of magnetic stars are constructed by the method of the magnetic charge distribution (MCD). The surface magnetic field is the linear summation of the vector components of the individual fields of virtual magnetic monopoles, which combine to magnetic dipoles and multipoles. The MCD-method relates to the construction of a vector field out of its sources and vortices, which is comfortable for progamming on a computer and possesses a wide range of applicability, e.g., for a decentered dipole or flat dipoles under the surface like sun spots.

A theoretical derivation is given for the calculation of the magnetic surface field and the entire Stokes vector. This yields the algorithms for drawing the magnetic map of the star on the base of the model.[1]

The **structure of the magnetic field** of a star on its surface is covered from observation by a lot of information destroying processes. For the reconstruction of the original surface distribution from the final observational values all these processes have to be inverted. In contrary to this, a straight-forward calculation can be carried out in any case. For this we give here the theoretical foundation on the MCD-method as outlined already in previous papers [3-9].

The **calculation** of magnetic fields in stars has a long history. We refer especially to Oetken [1], who modeled the star as an equatorially symmetric rotator. Krause and Rädler [2] treated the stellar magnetism by the action of an electromagnetic dynamo.
We use as generators for the magnetic field ***virtual magnetic charges***.

The magnetic field is a **vector field**, which is defined completely by its *wells and whirls* – or scientifically called: **sources and vortices**. The MCD-method relates only to the *virtual magnetic sources* using the algorithm for a monopole. With $\boldsymbol{B}$ as the field vector, the absence of sources is expressed by the relation div $\boldsymbol{B} = \mathbf{0}$. The magnetic field lines are closed and have neither a beginning nor an ending point, i.e., magnetic monopoles do not exist in reality.

---

[1] Submitted to: Proceedings of a workshop on *Magnetic Fields across the Hertzsprung-Russel Diagram.* IAU Conference in Santiago, Chile, 15-19 January 2001, ASP Conference Series, 2001, Vol. 248, p. 333-336
eds. G. Mathys, S.K. Solanki, and D.T. Wickramsinghe. (Preprint: E. Gerth)
Publication quoted by Harvard: `http://adsabs.harvard.edu/full/2001ASPC..248..333G`

However, a magnetic dipole, like an electric dipole, consists of two charges of opposite sign, which are firmly coupled with a *magnetic moment*

$$\boldsymbol{M} = Q\,\boldsymbol{l}\,, \tag{1}$$

where Q is the "magnetic charge" and $\boldsymbol{l}$ is the length difference of the dipole center between the two charge locations. Thus the magnetic moment is a vector and undergoes all rules of vector algebra. This has the consequence, that the virtual magnetic charges of dipoles and multipoles may formally be treated like separated individual field sources with arbitrary spatial distribution – provided the the sum of all charges is zero. The treatment of the ***magnetic charges*** as individual and separated field sources has an important advantage: the location of a charge is determined by its 3 spatial coordinates. But now we have to distinguish between ***poles and charges***.

If the charge is displaced from the center of the star, the polar coordinates (r radius, φ longitude, δ latitude) determine its location. Then by transformation to Cartesian coordinates

$$\begin{aligned} x &= r\cos\delta\cos\varphi \\ y &= r\cos\delta\sin\varphi \\ z &= r\sin\delta \end{aligned} \tag{2}$$

the distance from the center to the i-th point of the source is (with $a = r/\mathrm{R}$ representing the fraction of the star's radius):

$$r^2 = R^2[(\cos\delta\cos\varphi - a_i\cos\delta_i\cos\varphi_i)^2 + (\cos\delta\sin\varphi - a_i\cos\delta_i\sin\varphi i)^2 + \\ + (\sin\delta - a_i\sin\delta_i)^2]. \tag{3}$$

The magnetic charge Q produces a ***central symmetric potential*** $U = Q/(4\pi\mathrm{R})$ with the radius R, from which the field strength is derived by the gradient

$$\boldsymbol{B} = -\,\mathrm{grad}\,U. \tag{4}$$

The gradient is a vector of 3 components, which span a space with 3 orthogonal unity vectors as Cartesian coordinates **i, j, k**. Then we have for each point of the sphere in the polar orthogonal system the gradient

$$\mathrm{grad}\,U = \frac{\partial U}{\partial r}\frac{\mathrm{d}r}{\mathrm{d}x}\mathbf{i} + \frac{\partial U}{\partial r}\frac{\mathrm{d}r}{\mathrm{d}y}\mathbf{j} + \frac{\partial U}{\partial r}\frac{\mathrm{d}r}{\mathrm{d}z}\mathbf{k}. \tag{5}$$

If we use for polar coordinates only the radius r and simplify the constant with the charge to $C = -Q/4\pi$, then the potential $U = -C/r$ yields $\mathrm{d}U/\mathrm{d}r = -C/r^2$. The differential quotients along the 3 orthogonal polar coordinates are:

$$\begin{aligned} \boldsymbol{B}_r &= \partial U/\partial r = (\mathrm{C}/r^3)[\cos\delta(\cos\varphi + \sin\varphi) + \sin\delta] \\ \boldsymbol{B}_\varphi &= \partial U/\partial\varphi = (a\mathrm{C}/r^3)\cos\delta(\cos\varphi - \sin\varphi) \\ \boldsymbol{B}_\delta &= \partial U/\partial\delta = (a\mathrm{C}/r^3)[\cos\delta - \sin\delta(\sin\varphi + \cos\varphi)] \end{aligned} \tag{6}$$

These equations are the basic relations for the calculation of the magnetic field strength distribution over the star's surface for a **single monopole**. The differential quotients represent the magnetic field vector at the surface of the star. The **mapping** of the magnetic surface structure relates to these values.

In the case of a **dipole**, a superposition of two monopole fields of opposite sign takes place. The summation of the fields of monopoles can arbitrarily be continued. Only, for magnetic fields the pair-like combination of magnetic charges has to be obeyed, because the sum of the charges must be zero.

The resultant values of the magnetic field strength distributed over the stellar surface represent the **map of the star** $B(\varphi, \delta)$. The globe of the star is seen by the observer under different aspects, caused by its rotation and the inclination i to the rotational axis. The visible disk with ε as the angle from its center is vignetted by the limb darkening according to the empirical formula

$$k = 1-0.4\cos\varepsilon \, . \tag{7}$$

For the visibility of the star by the observer we define a **window function** $W(i, \varepsilon, \delta, \varphi)$, containing the inclination i, the projection of each surface element on the line of sight, and the limb darkening parameter ε. $W(i, \varepsilon, \delta, \varphi)$ averages and normalizes the magnetic map distribution function $B(\varphi, \delta)$:

$$\boldsymbol{B}_{\text{eff}}(t) = \frac{\int\limits_{\delta=-\pi/2}^{\pi/2} \int\limits_{\varphi=0}^{2\pi} B(\delta,\varphi) W(i,\varepsilon,\delta,\varphi-t) \mathrm{d}\varphi \mathrm{d}\delta}{\int\limits_{\delta=-\pi/2}^{\pi/2} \int\limits_{\varphi=0}^{2\pi} W(i,\varepsilon,\delta,\varphi-t) \mathrm{d}\varphi \mathrm{d}\delta} \tag{8}$$

This is the general relation between the magnetic field map and the observable **integral radiation flux** over the visible stellar surface, which we use especially for the magnetic field with all its vector components. The integral formula gives the integral mean of the disk seen by the observer and comprises the **convolution integral**, which represents the rotation of the star with its map $B(\varphi, \delta)$ behind the window $W(i, \varepsilon, \delta, \varphi)$. The variable t characterizes the rotation at the time of the momentary orientation angle. For the numerical calculation by a computer, we replace the integral transformations by **matrix multiplication**. The map is discretized into surface areas as matrix elements, each element representing the integral mean value of this area.

The calculation of the magnetic field strength yields a triple of values to every point of the stellar surface, the visibility of which depends on the **conditions** bound to **geometry, phase and physics** of the star. The magnetic field strength, which the observer measures by the Zeeman splitting of spectral lines, is the result of

1. projection by the coordinate orientation,
2. weighting by different areas of the elements and
3. vignetting by limb-darkening.

The computation relates to the fact, that the gravity center of the line profiles of different height and position is given by the mean of the centers weighted by the profile integrals. Thus, we weight the magnetic field vector, projected onto the line of sight, of all surface elements with their area size, spherical projection and limb darkening and integrate them over the visible hemisphere. The radial direction of the field vector in every element on the surface is given in Cartesian coordinates with the unity vectors **i, j, k** and the geographical coordinates of the longitude φ and the latitude δ to

$$\mathbf{a}_r = \mathbf{a} = \cos\delta\cos\varphi\mathbf{i}+\cos\delta\sin\varphi\mathbf{j}+\sin\delta\mathbf{k} \, . \tag{9}$$

The two orthogonal tangential directions to the normal direction are:

$$\mathbf{a}_\varphi = \partial\mathbf{a}/\partial\varphi = -\cos\delta\sin\varphi\mathbf{i}+\cos\delta\cos\varphi\mathbf{j} \tag{10}$$

$$\mathbf{a}_\delta = \partial\mathbf{a}/\partial\delta = -\sin\delta\cos\varphi\mathbf{i}-\sin\delta\sin\varphi\mathbf{j}+\cos\delta\mathbf{k} \tag{11}$$

With the 3 spherical components B_r, B_φ, and B_δ, the magnetic field vector at the surface of the star is given in Cartesian coordinates:

$$\boldsymbol{B} = B_r\,\mathbf{a}_r + B_\varphi\,\mathbf{a}_\varphi + B_\delta\,\mathbf{a}_\delta \tag{12}$$

The magnetic field components are seen by the observer from a special aspect projected onto the line of sight, which we denote as the vector $\mathbf{v}$:

$$\mathbf{v} = \sin i\cos t\mathbf{i} + \sin i\sin t\mathbf{j} - \cos i\mathbf{k} \tag{13}$$

The **projection** of the magnetic field vector $\boldsymbol{B}$ related to each point of the surface is carried out by a **scalar multiplication** adjusted to the vector of the line of sight $\mathbf{v}$

$$B_v = \boldsymbol{B}\ \mathbf{v} = [B_r\,\mathbf{a}_r + B_\varphi\,\mathbf{a}_\varphi + B_\delta\,\mathbf{a}_\delta\,]\,\mathbf{v}, \tag{14}$$

which is the component **V** of the ***Stokes vector*** by circularly polarized light:

$$\begin{aligned} B_v = \boldsymbol{B}\ \mathbf{v} = {} & B_r[\cos\delta\sin i(\cos\varphi\cos t + \sin\varphi\sin t) - \sin\delta\cos i] + \\ & + B_\varphi[\cos\delta\sin i(\cos\varphi\sin t - \sin\varphi\cos t)] + \\ & + B_\delta[-\sin\delta\sin i(\cos\varphi\cos t + \sin\varphi\sin t) - \cos\delta\cos i] \end{aligned} \tag{15}$$

Likewise, we can calculate the projection onto the plane perpendicularly to the line of sight, as the scalar products of the two orthogonal directions to the vector $\mathbf{v}$. The 2 perpendicular vectors to $\mathbf{v}$ are found by permutation of the unity vectors **i, j, k**:

$$\mathbf{q} = \sin i\sin t\mathbf{i} - \cos i\mathbf{j} + \sin i\cos t\mathbf{k} \tag{16}$$

$$\mathbf{u} = -\cos i\mathbf{i} + \sin i\sin t\mathbf{j} + \sin i\cos t\mathbf{k} \tag{17}$$

Thus, the field components of the linear polarization, namely the STOKES parameters **Q** and **U**, are derived by scalar multiplication:

$$\begin{aligned} B_q = \boldsymbol{B}\ \mathbf{q} = {} & B_r(\cos\delta\cos\varphi\sin i\sin t - \cos\delta\sin\varphi\cos i + \sin\delta\sin i\cos t) + \\ & + B_\varphi(-\cos\delta\sin\varphi\sin i\sin t - \cos\delta\cos\varphi\cos i) + \qquad (18) \\ & + B_\delta(-\sin\delta\cos\varphi\sin i\sin t + \sin\delta\sin\varphi\cos i + \cos\delta\sin i\cos t) \end{aligned}$$

$$\begin{aligned} B_u = \boldsymbol{B}\ \mathbf{u} = {} & B_r(-\cos\delta\cos\varphi\cos i + \cos\delta\sin\varphi\sin i\sin t + \sin\delta\sin i\cos t) + \\ & + B_\varphi(\cos\delta\sin\varphi\cos i + \cos\delta\cos\varphi\sin i\sin t) + \\ & + B_\delta(\sin\delta\cos\varphi\cos i - \sin\delta\sin\varphi\sin i\sin t + \cos\delta\sin i\cos t) \end{aligned} \tag{19}$$

The components B_q, B_u, and B_v represent 3 of the 4 STOKES parameters. The parameter **I** is the mean field strength as the square root of the intensity:

$$B_{mean} = \sqrt{B_q^2 + B_u^2 + B_v^2} \tag{20}$$

If we take out of equation (20) only the **Q** and **U** components,

$$B_{cross} = \sqrt{B_q^2 + B_u^2}\,, \tag{21}$$

then the plane of the crossed linear polarization perpendicularly to the line of sight comes in view. B_{cross} multiplied with the projected rotational velocity v sin i is an observable magnitude (BAGNULO [10]: "crossover").

As an example for magnetic modeling and mapping of a real star we give the calculated distribution of the surface magnetic field of HD 37776 in [9].[2])

---

[2]) Publication quoted by Harvard: `http://adsabs.harvard.edu/full/2001ASPC..248..337G`

The algorithms for modeling and mapping of magnetic fields have been realized in a computer program written by E. GERTH.
Fig. 1 shows the *Mercator map* and the globe of a magnetic monopole.

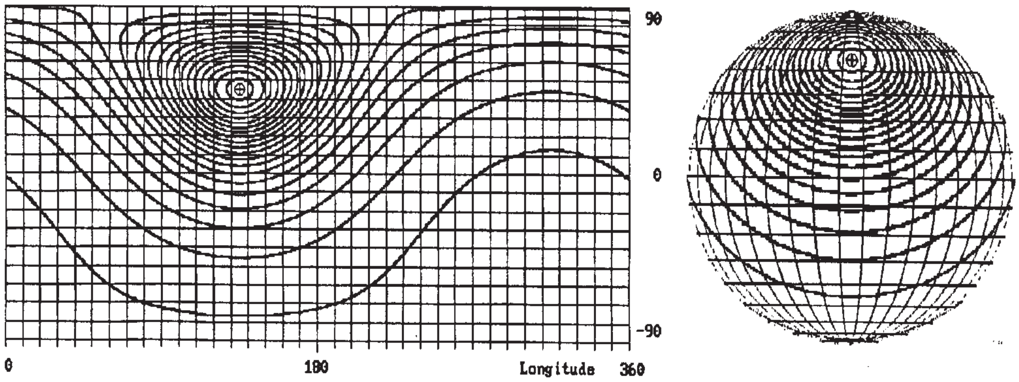

Figure 1. *Mercator − map* and spherical projection (globe) of a positively charged monopole ⊕.
The *iso − magnetic* lines mark the topographic location of equal magnetic field strength.

1. Charge	$Q = 1$	(relative units)	3. Longitude	$\varphi = 135^o$
2. Radius	$r = 0.5$	(fraction of the radius)	4. Latitude	$\delta = 45^o$

Further examples of stellar magnetic mapping are demonstrated in a review (Vienna, Workshop on Magnetic Stars, 2007) over ten years since the beginning of the *Magnetic Charge Method*.[3]

References

[1] Oetken, L., 1977, Astron. Nachr., **298**, 197; Oetken, L., 1979, Astron. Nachr., **300**, 1

[2] Krause, F. and Rädler, K.-H., 1980, Mean-Field Magnetohydrodynamics and Dynamo Theory, Akademie-Verl., Berlin, and Pergamon Press, Oxford

[3] Gerth, E., Glagolevskij Yu. V., Scholz, G. 1997, Integral Representation of the Surface Structure of the Stellar Magnetic Field, in Stellar Magnetic Fields, eds. Yu.V. Glagolevskij and Romanyuk, Moscow 1997, 67

[4] Gerth, E., Glagolevskij Yu.V., Scholz, G., 1998, Contr. Astr. Obs. Skalnaté Pleso, **27**, 455

[5] Glagolevskij, Yu. V., Gerth, E, Magnetic Models of ε UMa and HD 147010. 1998, Bull. Spec. Astrophys. Obs., **46**, 123

[6] Glagolevskij, Yu. V., Gerth, E., Hildebrandt, G., Lehmann, H., Scholz, G., 1998, Contr. Astron. Obs. Skalnaté Pleso, **27**, 458

[7] Glagolevskij, Yu. V., Gerth, E., Hildebrandt, G., Scholz, G., 1998, Contr. Astron. Obs. Skalnaté Pleso, **27**, 461

[8] Gerth, E., Glagolevskij, Yu. V., Hildebrandt, G., Lehmann, H., Scholz, G., 1999, A&A **351**, 133

[9] Glagolevskij, Yu. V., Gerth, E.: The magnetic model of HD 37776; in press.

[3]Lecture (with slides) at the conference in Vienna 2007: `www.ewald-gerth.de/123lec.pdf`
Article: `http://www.astro.sk/caosp/Eedition/FullTexts/vol38no2/pp173-178.pdf`

[10] Bagnulo, S.: 1998, Contr. Astron. Obs. Skalnaté Pleso, **27**, 431