Integral Representation

of the

Stellar Surface Structure

of the

Magnetic Field

Gerth, E, Glagolevskij, Yu, V., Scholz, G.

The **surface structure of the magnetic field** in stars is an important information of the surface inhomogeneities among others like brightness, movement or chemical composition, which is contained in the integral radiation flux but distorted beyond recognition by losses of information of the topographical arrangement over the surface because of convolution processes and partial invisibility.
The **simulation of the complicated processes**, beginning with the representation of the map of the surface field distribution and ending with the resulting phase-curve of the integral magnetic field, is the achievement performed with the computer program we present here.

General problems of mapping

Mapping of the topographical arrangement of individual features on the surface of a star out of the integral radiation can be carried out only in the case that the linear convolution integral transform leading to the integral flux is complete and can be inverted. This exceptional case is given if we look at the rotational equator of the star. The normal case is the inclined aspect making the correct deconvolution on principle impossible. We call this the ill-posed inverse problem, which cannot be solved correctly.
The precondition for the inversion are linear relations. Nonlinear processes due to physical processes in the stellar atmosphere are not accounted for.

A suitable way to tackle the **inverse problem** is the restriction to a small number of essential parameters describing the physical, topographical and geometrical conditions. The integral flux will be calculated by a straight-forward computation varying the parameters. The least-square deviation of the computed results to the observed ones give a criterion of the quality of approximation, which could be used in an iteration cycle.
However, such a method does not give an unambiguous result. Different constellations of parameters can yield similar results. The **iteration** depends strongly on the initial parameters and might end in different potential hollows of statistical entropy. The results have only a limited probability and therefore reliability. Additional information - for instance, by further observations - can clear up the situation or could twist the result to its contrary.

One way to overcome the ambiguity of mapping by means of solving the inverse problem is the **completion of information by further observation**. However, the invisible parts of the star remain hopelessly unknown forever.
The other way is the **involvement of the skilled astrophysicist**, who gives the direction of all further calculations and evaluations on the basis of his growing experience and should not be replaced at all by the authority of the computer.

For this purpose the **astrophysicist needs a tool**, which he can use to calculate the integral flux containing the physical magnitude - here the magnetic field strength - taking as a basis arbitrarily chosen hypothetical parameters to see quickly, what will be the outcome. This tool will be the **program of magnetic mapping** we present here.

Programming of the map

The map of **the topographical arrangement** of the magnetic field with its vectorial character on the surface of the star body is constructed consequently by matrices. The matrix elements are defined by the usual spherical coordinates of the longitude and the latitude. By this way of cartographical projection as a rectangular matrix, the areas of the elements become narrower from the equator to the poles. Respectively, all topographical structures in this representation of the map appear broader in direction to the poles.

The **magnetic field vector** consists of three components with the unity vectors in direction of the radius of the star (normal vector), in direction of the longitude (fi-vector), and in direction of the latitude (delta-vector). The components are stored together in a matrix on the hard disk. A fourth component is added for a scalar magnitude, which can be used for different purposes (brightness, transparency, factor). A reserve for two further quantities is foreseen (chemical composition, movement).

The calculation of the magnetic field components makes use of the fact that the linear aggregates of the potentials of **point-like field sources** are superposed linearly, which holds also for the derivatives. Thus, the potential of a single source will be calculated by the transform of rectangular to spherical coordinates. Then the field vector is easily derived by the spherical gradient of the potential.

The advantage of the **linear superposition of potentials and vectorial fields** is obvious. The calculation is not limited to special source configurations, say to the dipole or the quadrupole, which formerly was derived by complicated analytical treatments, e.g. using Legendre functions. The individual treatment of monopoles allows an arbitrary composition of configurations up to higher multipoles. In principle, any field you like can be represented by a row of monopole fields (a well-known lemma of the potential theory). On physical grounds the sum of positive and negative magnetic sources should be zero. But the program endures also a surplus of any polarity as we have this in the case of electrical charges.

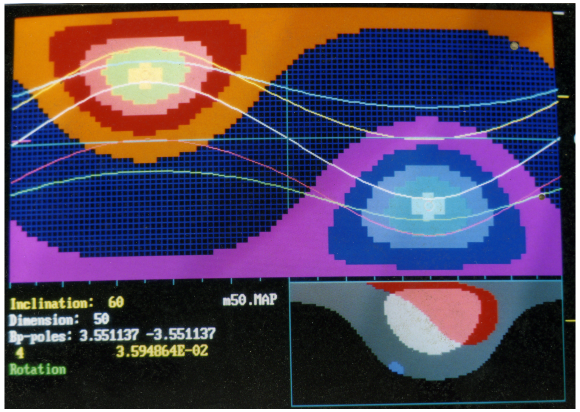
The **arrangement of the sources** may be anywhere in the interior of the stellar body, determined by spherical coordinates with a fraction of the stellar radius. A positive and a negative source make something like a rod magnet, leaving open what might be physically the connection line between them, maybe a magnetic tube. Easily there could be placed magnetic sources at the surface, representing stellar spots like sun spots. Sources outside the star could be positioned in companions, which influence the surface of the main star with their fields.

Aspect window and convolution

The **aspect window** is determined by the visible hemisphere of the star. The program computes for every chosen inclination angle in respect to the rotation axis the projection of the elements and the limb darkening comprising it to a rectangular matrix of the same rank as the map.

The matrices of the map and of the aspect window are subjected to a **matrix convolution**, which corresponds to the rotation of the star inclined to the line of sight to the observer. However, the "rotation" should not be understood merely as a process running in time. It is rather a series of geometrical aspects with a number of steps, determined by the rank of the matrix.

The **convolution algorithm** is the core of the program. It is multivalent and can be used also for other surface magnitudes. The demand of computing time depends quadratically on the rank of the matrix. In the present version of the program the highest rank is 98.



Informations to the pictures:

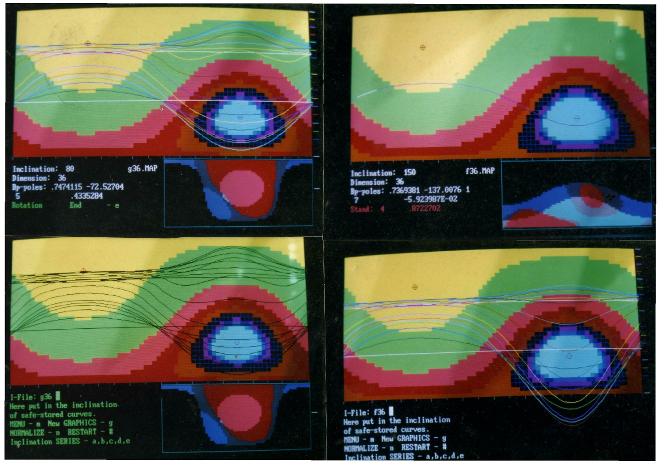
2 magnetic poles are arranged in all cases:

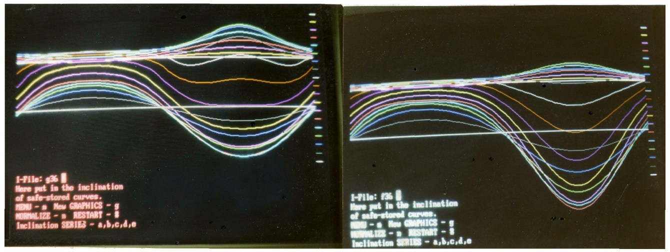
	Pole strength	Longitude	Latitude
1.	+1	90°	45°
2.	-1	270°	-45°

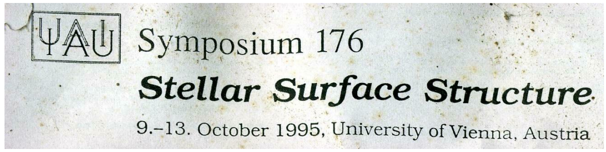
The depth of the poles is given by the fraction of the radius:

map	rank	+ pole	- pole
m50	50	0.5	0.5
c36	36	0.5	0.5
f36	36	0	0.95
g36	36	0	0.99

In the last cases the south pole lies in the center of the star.
The north pole has a different depth under the surface.
The dip in the curve occurs only if a pole is arranged near the surface.







Physical problems of the integral magnetic field

Usually we interpret the (effective) stellar magnetic field by the **Zeeman displacement** of the gravity centers of the line profiles of oppositely polarized light. Since the light comes from the visible hemisphere of the star, all parts of the surface contribute differently to the whole profile and its resulting position. What is called the "effective magnetic field" is not a mean value but already the result of weighting and convolution of the radiation flux containing the magnetic field information in the form and position of the profiles of all surface elements.

The program allows the distinction between different types of polarization and their physical significance. Therefore, the transmission of the **flux through the atmosphere** has to be treated correctly by the methods of the radiation transfer theory using the four Stokes parameters. The program is open for introduction of the transfer theory, which will be done in the future.

Nevertheless, **overcharging** with time-consuming procedures should be avoided as far as possible. The polarization problem can be managed with the magnetic vector field alone.

For this we relate to the fact, that the **gravity center** of two profiles of different height and position is given by the mean of the centers weighted by the profile integrals. Thus, we weigh the magnetic field vector of all surface elements with their spherical projection onto the line of sight including the limb darkening and integrate them over the visible hemisphere. That means, we use for the magnetic field the same procedure as for the brightness, regarding only the vectorial character of the magnetic field.

Graphical representation and storage of maps and the results

The usefulness of the graphical representation of the calculation is beyond any discussion. However, in the case of the present program, **computer graphics** have already served at the first stages of development for controlling and testing purposes, so that it is hard to imagine how to manage without it.

At first the construction of the map has been programmed for the **brightness distribution**. The algorithm has later been taken over for the vector components. The colors are used to mark a scale of magnitude steps. The map gives a good overlook of the distribution of the physical magnitude over the star's surface in a rectangular cartographical projection.

The **rotation curves** are computed and drawn on to the map, so that it is easy to see how the topographical features act onto the curves. Different curves with the parameter of the inclination angle are distinguished by colors.

The **movement of the poles and spots** over the visible surface by rotation of the star is shown in a special graphics. These images help us to understand why there occur maxima and minima of the curves.

The calculated **maps are stored** as files on disk and may be recalled repeatedly for different calculations and graphical representations in connection with the resulting phase curves.

The **stored resulting curves** can be recalled from the disk and represented together in the graphics in groups, which have been computed in a cycle each after the other. By this way also computation times at night could be used practically. The results can be copied to other carriers for archiving.

The use of the program

The program will be used for the **investigation** of special **forms of curves** as having been observed in magnetic stars.

A **catalogue of maps and resulting curves** with systematic variation of parameters will be comprised in a data bank for comparison with observed curves.

The program will be **applied at real early type magnetic stars** in order to determine the topographical positions of the poles and to discover their magnetic structure as the basis for the explanation of the connection between stellar magnetism and chemical composition.

First results

A lot of maps of different arrangements of poles have already been calculated, giving some **feeling** about the influence of the parameters. For instance:

1. Very interesting is the phenomenon, that a pole on or near the surface of the star produces a **dip** in the rotation curve. This was at first surprising but could easily be understood: The bending of the field lines from the pole back to the star center **reverses the direction** of the lines penetrating the surface for a large visible area around the pole.

2. Curves of pole arrangements of equal but opposite charges with one pole near the surface and the other deeper in the interior of the star show a **surplus of one polarity**, which seems to violate the law of div $\boldsymbol{B} = 0$. But this is also connected with the integral of the flux over the visible parts of the star.

3. Regarding a map of a factor like transparency connected with the chemical distribution, the original magnetic curve will be very much **distorted** so that no conclusion about the magnetic structure could be drawn. But now we can investigate the influence of such a factor.

4. Some **examples of the graphical representation** are shown here to demonstrate how the programm works.

Testing and further development

The program is still in the **testing phase**. All procedures have to be proved thoroughly. However, the established correct function could be regarded as the most important first result.

The program is **menu-guided**. The menus give the necessary hints and explanations.

A **description of the program** and its use is in preparation. The explanations will be called by the help-command.

After the testing phase the main algorithm of the program will be included into an **iteration cycle** for tackling the inverse problem.

In any case: The development of the magnetic mapping program goes on.

Some remarks to the reconstruction of the poster from 1995 in 2007:

The poster has been restored after the preserved original rather accurately.
The text and the photographic pictures are scanned and completely inserted.
That time (1995) the pictures were taken photographically from the screen of a monitor.
The emblem on the top left was given and fastened to the poster during the conference.
The poster was presented on the poster-session, but it was not included in the conference volume, presumably because the application was a last minute one, so that it was not registered any more. Nevertheless, we regard it worthwhile, to look at the first document of the construction of a stellar magnetic field out of its sources, which shows already in the beginning the broad applicability of a computer program for the straightforward calculation.
This *not published* poster is available from the *homepage* by the address
www.ewald-gerth.de/90pos.pdf

Poster representation at the **IAU Symposium 176** on ***Stellar Surface Structure*** in Vienna, Austria, October 9-13, 1995 (not published)