Integral representation of the surface structure of the stellar magnetic field

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Abstract. The surface structure of the magnetic field in stars is one of several types of information of the surface inhomogeneities such as brightness, movement or chemical composition, which is contained in the integral radiation flux but distorted beyond recognition by losses of information of the topographical arrangement over the surface because of convolution processes and partial invisibility.

The simulation of the complicated processes, beginning with the representation of the map of the surface field distribution and ending in the resulting phase–curve of the integral magnetic field, is the advance made with the computer program we present here.

1 General problems of mapping

Mapping the topographical arrangement of individual features on the surface of a star from the integral radiation can be carried out only providing the linear convolution integral transform leading to the integral flux is complete and can be inverted. This exceptional case is given if we look at the rotational equator of the star. The normal case is the inclined aspect making the correct deconvolution basically impossible. We call this the ill–posed inverse problem, which cannot be solved correctly.

The preconditions for the inversion are linear relations. Nonlinear processes due to physical processes in the stellar atmosphere could not be accounted for.

A suitable way to tackle the inverse problem is the restriction to the small number of essential parameters describing the physical, topographical and geometrical conditions. The integral flux will be calculated by a straight–forward calculation varying the parameters. The least-squares deviation of the computed results from the observed ones give a criterion of the quality of approximation, which could be used in an iteration cycle.

However, such a method does not give an unambiguous result. Different constellations of parameters can yield similar results. The iteration depends strongly on the initial parameters and might end in different potential hollows of statistical entropy. The results have only a limited probability and therefore reliability. Additional information — for instance, from further observations — can clear up the situation or could twist the result to its contrary.

One way to overcome the ambiguity of mapping by means of solving the inverse problem is the completion of information by further observation. However, the invisible parts of the star remain hopelessly unknown forever. The other way is the involvement of a skilled astrophysicist, who would give the direction of all further calculations and evaluations on the basis of his growing experience and should not be replaced at all by the authority of the computer.

For this purpose the astrophysicist needs a tool, which he can use to calculate the integral flux containing the physical quantity — here the magnetic field strength — taking as a basis arbitrarily chosen hypothetical parameters to see quickly, what will be the outcome. This tool will be the program of magnetic mapping we present here.

2 Programming of the map

The map of the topographical arrangement of the magnetic field with its vectorial character on the surface of the star body is constructed consequently by matrices. The matrix elements are defined by the usual spherical coordinates of the longitude and the latitude. By this way of cartographical projection as a rectangular matrix the areas of the elements become narrower from the equator to the poles. Accordingly, all topographical structures in this representation of the map appear broader in the direction of the poles.

The magnetic field vector consists of three components with the unity vectors in the direction of the radius of the star (normal vector), in the direction of the longitude (ϕ vector), and in the direction of the latitude (δ vector). The components are stored together in a matrix on hard disk. A fourth component is added for a scalar magnitude, which can be used for different purposes (brightness, transparency, factor). The calculation of the magnetic field components makes use of the fact that the linear aggregates of the potentials of point–like field sources are superposed linearly, which holds also for the derivatives. Thus, the potential of a single source will be calculated by the transform of rectangular to spherical coordinates. Then the field vector is easily derived from the spherical gradient of the potential.

The advantage of the linear superposition of potentials and vectorial fields is obvious. The calculation is not limited to special source configurations, say, to dipole or quadrupole, which formerly was derived from complicated analytical treatments, e.g. using Legendre functions. An individual treatment of monopoles allows an arbitrary composition of configurations up to higher multipoles. In principle, any field you like can be represented by a row of monopole fields (a well–known lemma of the potential theory). On physical grounds the sum of positive and negative magnetic sources must be zero. But the program endures also a surplus of any polarity as we have this in the case of electrical charges.

The sources may be arranged anywhere in the interior of the star body, determined by spherical coordinates with a fraction of the star radius. A positive and a negative sources make something like a rod magnet, leaving open what might be physically the connection line between them, maybe a magnetic tube. Easily there could be placed magnetic sources at the surface, representing stellar spots like Sun spots. Sources outside the star could be positioned in companions, whose field affects the surface of the main star.

3 Aspect window and convolution

The aspect window is determined by the visible hemisphere of the star. The program computes for every chosen inclination angle in respect to the rotation axis the projection of the elements and the limb darkening, comprising it, to a rectangular matrix of the same rank as the map. The matrices of the map and of the aspect window are subject to matrix convolution, which corresponds to the rotation of the star inclined to the line of sight. However, the "rotation" should not be understood merely as a process running in time. It is rather a series of geometrical aspects with a number of steps, determined by the rank of the matrix. The convolution algorithm is the core of the program. It is multivalent and can be used also for other surface magnitudes. The demand for computing time is square dependent on the rank of the matrix. In the present version of the program the highest rank is 104.

4 Physical problems of the integral magnetic field

Usually we measure the (effective) stellar magnetic field from the Zeeman displacement of the gravity centers of the line profiles of oppositely polarized light. Since light comes from the visible hemisphere of the star, all parts of the surface contribute differently to the whole profile and its resulting position. What we call the "effective magnetic field" is not a mean value but already the result of weighting and convolution of the radiation flux containing the magnetic field information about the form and position of the profiles of all surface elements. The transmission of the flux through the atmosphere has to be treated correctly by the methods of radiation transfer theory, using the four Stokes parameters. The program is open for introduction of the transfer theory, which will be done in future. But this would render the program overcharged with time–consuming procedures.

In this program we relate to the fact, that the gravity center of two profiles of different height and position is given by the mean of the centers weighted by the profile integrals. Thus, we weight the magnetic field vector, projected onto the line of sight, of all surface elements with their spherical projection and limb darkening and integrate them over the visible hemisphere. That means, we use for the magnetic field the same procedure as for the brightness, regarding only the vectorial character of the magnetic field.

5 The use of the program

The program will already be used in the next investigation of occurrence coming about of special shapes of curves, as have been observed in magnetic stars.

A catalogue of maps and resulting curves with systematic variation of parameters will be comprised in a data bank for comparison with observed curves.

The program will be applied to real early type magnetic stars in order to determine the topographical positions of the poles and to discover their magnetic structure as a basis for explanation of connection between stellar magnetism and chemical composition.

6 First results

The program is still at the testing phase. All procedures have to be proved thoroughly. However, the established correct function is already the most important first result.

6.1 53 Cam

In Fig.1 are presented the observed and calculated magnetic field and photometric variability versus the rotation period for the star 53 Cam (magnetic field has been measured (Borra, Landstreet, 1977), while the photometric variability has been studied (Preston and Stepien, 1968; Stepien, 1978)). The positive half–wave is a little wider than negative and may be explained by the dipole–quadrupole structure of the magnetic field. The solid line is the curve calculated under the assumption of central dipole, while the dashed line is the plot of the results for the dipole–quadrupole structure of the magnetic field. The dipole has the following parameters (the magnetic field strength at the star's surface):

No.	Monopole, kG	Longitude	Latitude
1	10	90°	10°
2	-10	270	-10

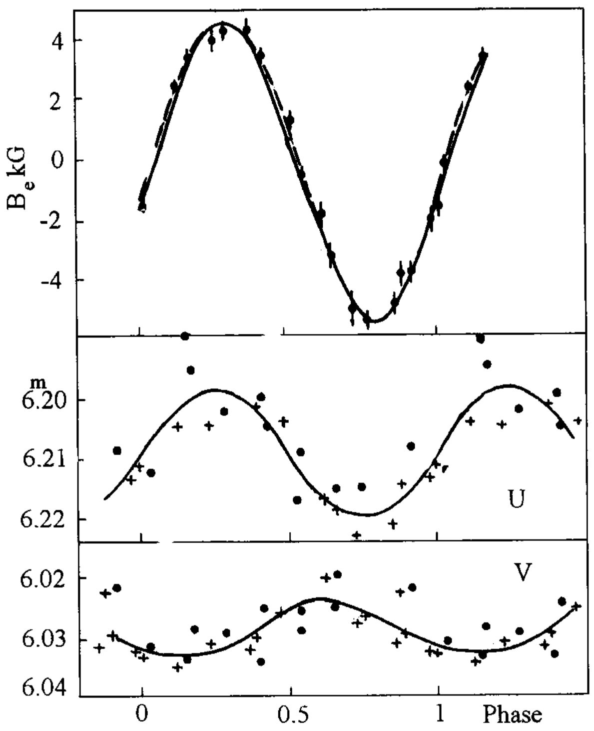

Figure 1: The effective longitudinal magnetic field B_e and photometric variability in the U and V filters versus the rotation phase for the star 53 Cam. The curves are plotted by the method of model magnetic and photometric variability. The solid line — the central dipole model; the dashed line — the dipole–quadrupole model. The photometric data: filled circles – Preston, Stepien (1968); + – Stepien (1978).

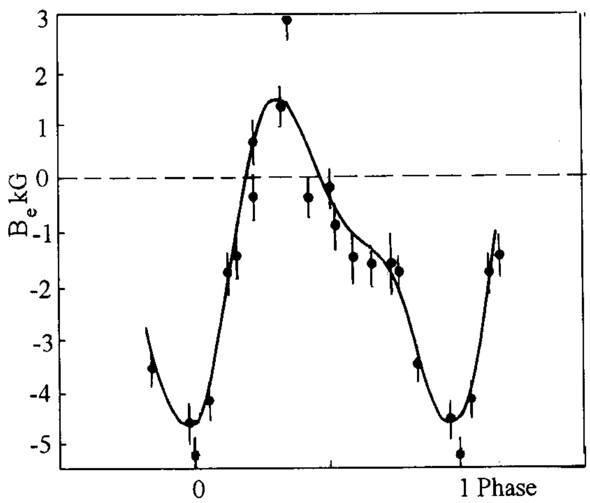

Figure 2: The symmetric model of the dipole–quadrupole magnetic field of HD 32633 corresponding to H-beta measurements of Borra and Landstreet (1980).

Inclination angle of the rotation axis is $i = 55°$.

The parameters of the dipole–quadrupole field are as follows:

No.	Monopole, kG	Longitude	Latitude
1	7	0°	0°
2	-7	90	10
3	7	180	0
4	-7	270	-10
5	12.5	90	10
6	-12.5	270	-10

Inclination angle of the rotation axis is $i = 64°$.

The dipole and quadrupole are located in one and the same plane. The magnetic sources are located at the distance $R/R_* = 0.1$ of radius from the center of the star.

The ratio of the quadrupole moment to the dipole moment $U_q/U_d = 0.56$, which is close to the result of Oetken (1979) for 53 Cam $U_q/U_d = 0.5$. ($i = 60°$, $\chi = 90°$).

The magnetic model of Landstreet (1988) has used dipole, quadrupole and octupole components and $i = 64°$, $\chi = 82°$. These results are excellently coincide with ours.

The photometric curves were obtained assuming that the region of the negative pole in the V–filter is 1% brighter than the positive pole region (with the region diameter equal to 140°), while in the U–filter it is by 2.5% weaker (with the diameter of 150°). The latitude of the magnetic field poles coincides with the latitude of the centers of the photometric "spots", however the longitude of the "spot" in U and V filters differs from the longitude of the poles by 20° and 70°, respectively. The results presented are not analyzed in detail but given for the principal capabilities of the modeling technique to be demonstrated.

6.2 HD 32633

The star HD 32633 has one of the complicated inharmonic curves of the magnetic field variations. It appears that the curve is excellently presented by the dipole–quadrupole model. In Fig.2 are shown the data measured by Borra and Landstreet (1980) with the hydrogen–line photoelectric polarimeter. The curve is the calculated variability under the assumption of the following model:

No.	Monopole, kG	Longitude	Latitude
1	-80	0°	-50°
2	80	90	0
3	-80	180	50
4	80	270	0
5	8	90	0
6	-8	270	0

The inclination angle of the rotation axis is $i = 30°$.

For presentation of the measurements of Preston and Stepien (1968) made by the photographic method (Fig.3), it is necessary to use the following dipole–quadrupole model:

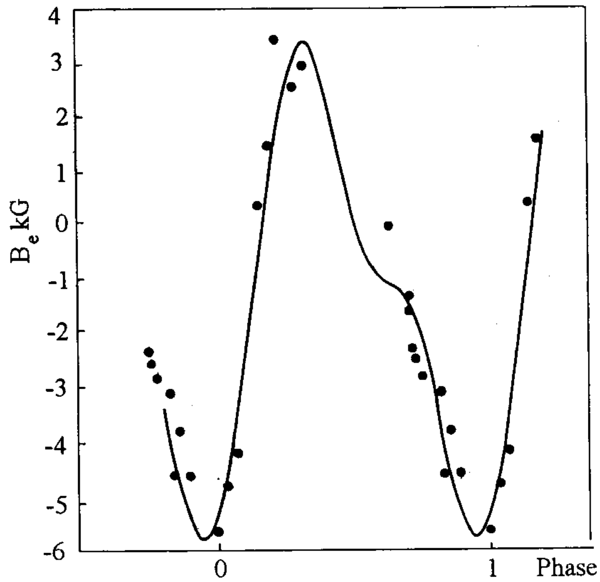

Figure 3: The symmetric model of the dipole–quadrupole magnetic field of HD 32633 corresponding to photographic measurements of Preston and Stepien (1968).

No.	Monopole, kG	Longitude	Latitude
1	-85	0°	-50°
2	85	90	0
3	-85	180	50
4	85	270	0
5	9	90	0
6	-9	270	0

The inclination angle of the rotation axis is $i = 35°$.

For all these examples we have used the position of two quadrupole monopoles coincident with the position of the dipole monopoles, but it appeared possible to improve the fit of the calculated and observed data provided that the position of some quadrupole monopoles is shifted with respect to the dipole monopoles. The result is shown in Fig.4 under the assumption of the following model:

No.	Pole, kG	Longitude	Latitude
1	-90	0°	-50°
2	90	90	0
3	-90	180	50
4	90	275	0
5	8	90	0
6	-8	270	0

The inclination angle of the rotation axis is $i = 36°$.

It can be seen that the longitude of one of the magnetic monopoles around 270° differs by 5°. The curve calculated under this assumption is consistent with the photographic observations of Babcock, published in (Renson, 1984), as it can be seen in Fig.5. All these data show that the monopoles lie in one and the same plane inclined to the rotation axis at an angle $\chi = 30°$.

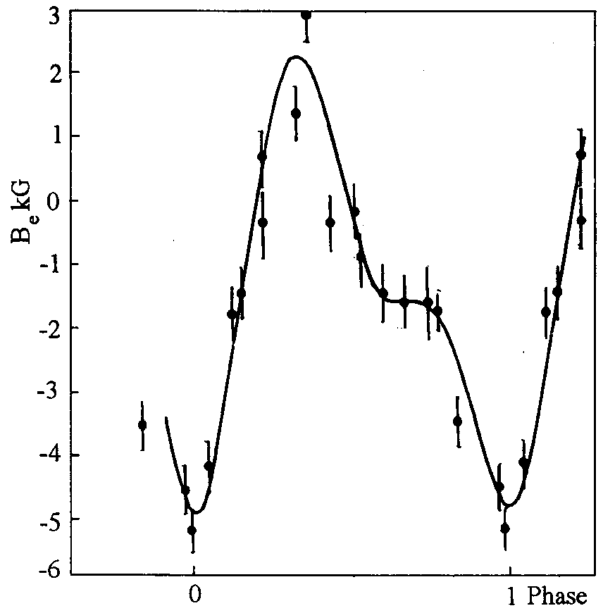

Figure 4: The asymmetric model of the dipole-quadrupole magnetic field of HD 32633 for the first measurements (Fig.2).

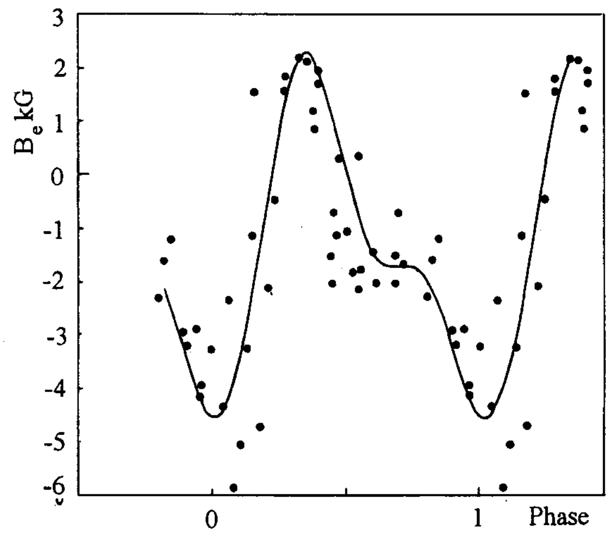

Figure 5: The same for the photographic measurements of Babcock (see the text).

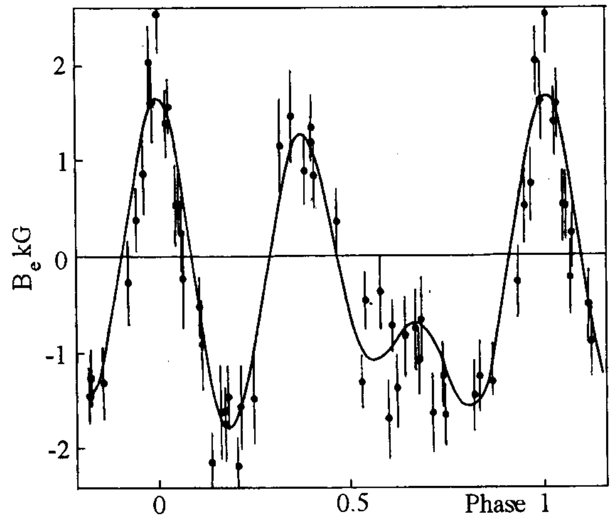

Figure 6: The asymmetric model of the dipole–quadrupole–sextupole magnetic field of HD 37776 corresponding to H-beta measurements of Thompson and Landstreet (1985).

6.5 HD 37776

The magnetic variability of this star is most complicated. The following sextupole–quadrupole–dipole model of magnetic field is the best fit to the observational data of Thompson and Landstreet (1985) (Fig.6):

No.	Monopole, kG	Longitude	Latitude
1	42	359°	10°
2	-42	57	4
3	42	118	-4
4	-42	181	-10
5	42	243	-4
6	-42	302	4
7	-7	75	2
8	7	172	-9
9	-7	262	-1
10	7	356	10
11	1.2	0	10
12	-1.2	180	-10

The inclination angle of rotation axis is $i = 82°$, the monopoles placed in the same plane are inclined to the rotation axis at an angle $\chi = 80°$, $R/R_* = 0.18$.

The distribution of magnetic monopoles for all samples calculated is shown in Fig.7. Note that the axes of dipoles and quadrupoles are coincident, one of the axis of sextupole also coincides with them. In all cases monopoles lie in the same plane. It can be assumed that such a symmetric distribution of monopoles may occur by sign that the magnetic field arises by dynamo.

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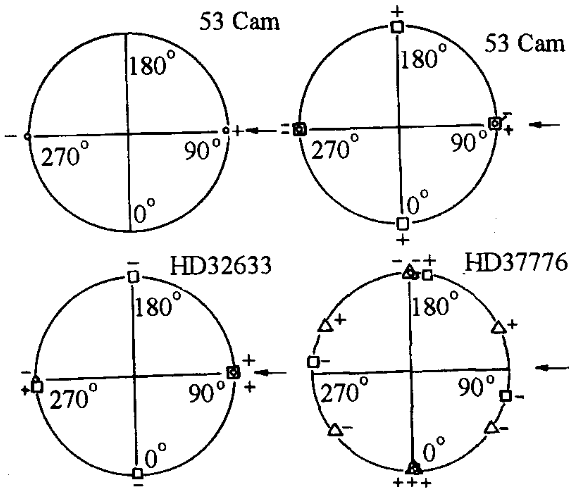

Figure 7: The magnetic models of 53 Cam (dipole and dipole+quadrupole), HD 32633 and HD 37776.

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