Derivation of a four-parameter density formula of the photographic characteristic curve

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Dedicated in his memory to KLAUS KRÖBER[1])

Summary

A simple four-parameter formula of the characteristic curve is derived on the basis of the theory of the photographic process, making use of special approximations that allow for elementary integration. The four parameters correspond to the saturation density, the opacity density, the step order of the development centers, and the sensitivity — representing the main quantities of the characteristic curve.
The formula may easily be inverted and it is suited for interpolation of the characteristic curve, being therefore appropriate for computerized sensitometric evaluation.

Zusammenfassung

Unter Anwendung spezieller Näherungen, die elementare Integrationen erlauben, wird eine einfache, vier-parametrige Formel für die Schwärzungskurve auf der Basis der Theorie des photographischen Prozesses hergeleitet. Die vier Parameter entsprechen der Sättigungsschwärzung, der Opazität der Schicht bei der Belichtung, der Reaktionsordnung der Entwicklungskeime und der Empfindlichkeit, womit die hauptsäch-lichen Größen der Schwärzungsfunktion dargestellt werden können.
Die Formel ist leicht invertierbar und eignet sich zur Interpolation der Schwärzungskurve, womit sie in einer rechnergestützten sensitometrischen Auswertung eingesetzt werden kann.

Резюме

Простая, с четырьмя параметрами, формула для представления кривой почернения выво-дится на основе теории фотографического процесса при использовании специальных аппроксимаций, позволяющих элементарное интегрирование. Этими четырьмя параметра-ми, которые представляют основные величины характеристической кривой, являются насыщенное почернение, оптическая плотность фотослоя при экспозиции, степень реакции проявительных центров и чувствительность.
Эта формула легко подвергается обращению и также пригодна для интерполяции кривой почернения, поэтому она рекомендуется для вычислительных сенситометрических обработок.

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[1]) Dipl.-Ing. K. KRÖBER died on Dec. 9th, 1988 at the age of 83. He was one of the leading engi-neers in the field of film techniques of the nationally owned movie-producing enterprise DEFA, engaged especially in the development of measuring and controlling methods for deriving quality standards of raw film materials. The author is deeply indebted to him for a long and true friend-ship as well as for a fruitful scientific cooperation, giving among other things the incentive to investigate the properties of the photographic emulsion represented by the characteristic curve.

0. Introduction

The "characteristic curve" is defined as the empirically determined and graphically represented functional relation between exposure and density resulting after development on a photographic layer, revealing the main characteristics of a photographic material alike a passport image. Between the exposure and the resulting density there is included the photographic process, which is veiled for direct insight by the nature of the light-sensitive object so that sensitometry always was regarded as a matter alone of empirism. Nevertheless, numerous proposals have been made for a mathematical formulation of the characteristic curve, as outlined in [11]; for example: the polynomial representation, the arctan-function, the GAUSS error integral, the BAKER function [1], and some more just phenomenological functions — all having actually no direct connection with the underlying physical process. Other mathematical treatments of the formulation of the characteristic curve base on the theory of the photographic process, which has been summarized in [11] and, recently, by ESCHRICH [2].

Despite the formula presented here can be found anywhere in both papers quoted last it seems to be favourable to a full comprehension of the significance of the four "characteristic parameters" to confine the derivation to the main roots, avoiding more redundancy — but without committing any loss of capability to generalization. Such a methodical variant of the derivation of the blackening formula has been given in a (rather rarely distributed) report [12] on an IAU-Workshop on Astrophotography 1987 in Jena (GDR), that in general we will follow in this paper, implementing some instructive completions.

1. Physically founded statements for the derivation of the characteristic function

The photographic process, as well known, is an extraordinarily complex one, being in this field of knowledge the main object of research hitherto pursued, so that it seems to be quite impossible to put all these results together in a single formula. Such a global description of the photographic process will probably never be achieved at all. Nevertheless, simplified models can be constructed, showing the main features in a general way, and thus allowing a better insight into the process, which is otherwise obscured by its own complexity.

Therefore, we group the considerations that influence the formation of the characteristic curve in their order of importance:

1. The microprocesses in the crystal lattice of the silver halide during the exposure
2. The grain size distribution
3. The transfer of light radiation through the emulsion layer during the exposure
4. The shape of the developed silver grains
5. The development and further treatments of the photolayer
6. The after-treatment of the developed layer

In the following the points 5 and 6 are not considered closer because they are not essential for producing the typical form of the characteristic curve.

2. The microprocesses in the silver halide crystal

Since microprocesses in the crystal lattice are not directly observable, a broad latitude is left for hypotheses and theories. Recent ideas on the photographic microprocesses are based on the theories of GURNEY and MOTT [13] with the completion by MITCHELL [15]

as summarized in [4]. In the present paper this wide field is not outlined. It should only be mentioned that in the past there turned out two main orientations of the theory:

1. the reaction-kinetics theory,
2. the statistical theory.

The first to attempt to describe the characteristic curve by means of reaction kinetics was SCHAUM [17] in 1909. The start of the statistical interpretation was made by SILBERSTEIN [18] in 1922. There is an increasing number of followers as outlined in [8]. Both branches of the theory describe the same object and are, in principle, equivalent. However, there are differences in describing the processing character of the creation of development centers, for any process, naturally, progresses in time. The statistical theory regards the statistics of photons and grains, but does not account for the time. Therefore, this theory does not give any explanation of SCHWARZSCHILD's law, which in the kinetics theory already emerges plausibly out of the basic statements [5], but can be derived exactly [7, 9].

3. Schwarzschild's law of blackening

As shown in [5] the product of power functions Et^p (E is intensity, t-time, p-SCHWARZSCHILD exponent) introduced by SCHWARZSCHILD as an independent variable of the density function can be interpreted as the result of a multi-step kinetic reaction of the build-up of latent image specks, which trigger the reduction of the whole silver halide grain to metallic silver as development centers. The explanation is given plausibly: The electron concentration c produced by the photoeffect in the lattice of the silver halide acts on each step of the reaction chain by discharging the center, resulting in a transition from one step to the higher one. Regarding the electron concentration c and the acting time t, the total probability for the transition of n steps producing a constant effect is the power product

$$c^n t^n = \text{const.} \tag{1}$$

If on one step — obviously the first — equilibrium is attained, then the time order reduces by one step, yielding

$$c^n t^{n-1} = \text{const.} \tag{2}$$

In exposure ranges where the electron concentration is proportional to the intensity (the long-time exposure, relevant in astrophotography), SCHWARZSCHILD's law

$$Et^p = \text{const} \tag{3}$$

follows with

$$p = 1 - 1/n. \tag{4}$$

In the case of 4 steps we have the SCHWARZSCHILD exponent $p = 0.75$. The quantity n is the reaction order of the centers representing the difference between the mean reaction order of the centers contained on all grains of the emulsion and the order (4) at which developability is reached. A method for the empirical determination of n is described in [14].
The photographic effect of the exposure depends on the SCHWARZSCHILD product

$$H = Et^p \tag{5}$$

as an independent variable called the effective light quantity. The average number $\bar{z}$ of centers per grain depends on the effective exposure H^n linearly with a factor of proportionality ε^*, which defines the sensitivity related to the developable grains,

$$\bar{z} = \varepsilon^* H^n. \tag{6}$$

Relating, however, the sensitivity to the intensity E acting on each reaction step, we may write instead of equation (6), using $\varepsilon^* = \varepsilon^n$,

$$\bar{z} = (\varepsilon H)^n, \tag{6a}$$

calling the quantities ε^* or ε the sensitivity coefficients. The effective light quantity H as being the result of the kinetic process of the build-up of development centers has to be calculated for correctness by an expense mathematical treatment using vectors, matrices, and tensors [10]. The SCHWARZSCHILD product equation (5) represents an ingenious simplification for a limiting case [9], which facilitates all mathematical derivations that imply the kinetical result of the exposure. Nevertheless, H may be replaced by a more general expression, either by the scalar product of exposure matrices and development speck-step vectors [7, 9] or, more simply, by a term derived from experimental data [11]

$$H = [(1 + aE)^b - 1] t^p, \tag{7}$$

which embraces the exposure regions of the normal and the inverse SCHWARZSCHILD-effect. In equation (7) a denotes a sensitivity coefficient and b an exponent, which emerges theoretically [5] to $b = 1/2$ but proves empirically to lie anywhere between the limits $1/2 \leq b \leq 1$. In the following the underlying expression for H is given only by equation (5).

4. The dependence of the sensitivity on the grain volume

The photoeffect affects the whole volume of the silver halide grain supplying the electrons needed for the build-up of development centers. So it is plausible but also experimentally proven that the sensitivity of a grain (in a wide range of small and average-sized grains) increases proportionally with its volume,

$$\varepsilon^* = \eta V, \tag{8}$$

where η denotes a coefficient for the sensitivity density and V the volume. Thus, the number of development centers can be written

$$z = \eta V H^n. \tag{9}$$

5. The relation between the area density of the grains and the optical density

The developed grains of a photographic layer are distributed statistically with respect to their sizes and positions. The POISSON probability $P(z)$ of the occupation z of one cell of the honeycomb-like imagined layer by grains is

$$P(z) = \frac{\bar{z}^z \exp(-\bar{z})}{z!} \tag{10}$$

with $\bar{z}$ denoting the mean occupation.
Since already one grain in the cell stops the light penetrating through the layer we have with $z = 0$ the transmission probability

$$P(0) = \exp(-\bar{z}), \tag{11}$$

called the transparency.
By definition of the density D as the decadic logarithm of the reciprocal transparency there follows

$$D = \log[1/\exp(-\bar{z})] = \bar{z} \log e. \tag{12}$$

Taking into account that the mean occupation $\bar{z}$ is given by multiplying the area density ϱ of grains by the projected mean squared area $\bar{a}$ of one grain, we obtain the well-known formula of NUTTING [16]

$$D = \varrho \bar{a} \log e \tag{13}$$

expressing a linear relation between density and area density of grains. Defining, however, the density by the natural logarithm as we do in the following throughout, the modulus $\log e$ will be left out, simplifying by this way most of the analytical expressions derived form any formula of the characteristic function. If the decadic logarithm is preferred, then the modulus $\log e$ or its reciprocal has to be introduced as a factor.

6. The relation between the occupation number of centers and the grain area density

The POISSON probability that a silver halide grain is capable of being developed can be written as

$$P_d = 1 - P(0) = 1 - \exp(-\bar{z}), \tag{14}$$

because at least one development center must be contained on the grain surface attainable to the developer.
Using the proportionality found by NUTTING we have with the equations (9) and (14), introducing further the saturation density D_∞, a first approach to the blackening function, yielding already the typical sigmoid curvature of the characteristic curve,

$$D(H) = D_\infty[1 - \exp(-\eta V H^n)], \tag{15}$$

which corresponds to the density formula of SVEDBERG [19].

7. The influence of the grain-size distribution

The silver halide crystals grow differently in the process of manufacturing the emulsion. The crystallization can be comprehended as a kinetic process of forward and back reactions, whereby the bigger crystals grow at the expense of the smaller ones. From experimental results it is known that the grain sizes have a scatter like a logarithmic normal distribution. It might be interesting, that such a distribution can be approximated by a sum of a few terms of exponential functions, which corresponds to the type of solution of reaction kinetics equations.
The bell-shaped curve may be achieved approximately by a distribution function $\omega(V)$ represented in the form of the difference of two exponential terms [11],

$$\omega(V) = \exp(-\alpha_1 V) - \exp(-\alpha_2 V). \tag{16}$$

However, it can be shown that the second term in equation (16) may be neglected, so that the one-term exponential function

$$\omega(V) = \exp(-\alpha V) \tag{17}$$

completely suffices.
The neglected term does not contribute considerably to the total density, because in the corresponding part of the distribution

1. the sensitivity of the grains is low,
2. the volume of the grains is small,
3. the share in the total number of grains is inconsiderable.

The form of the characteristic curve is determined essentially by the large grains. Only in the region of the shoulder do the small grains become considerable. The influence of the grain-size distribution may be formulated by the integral

$$D(H) = D_\infty \left[1 - N \int_0^\infty V^\varkappa \omega(V) \exp\left(-\eta V H^n\right) \, dV \right] \tag{18}$$

representing the summation of the contribution to the density distribution.

In (18) N is a normalizing factor, and $\varkappa$ denotes the "form exponent" depending on the shape of the developed silver grains.

In the case of a spherical volume the density contribution of a grain is proportional to the projection area, i. e. $V^{2/3}$.

Such silver grains are produced by half-physical development condensating the entire silver of the grain at the development center. In the extreme case of physical development all silver is precipitated out of the solution phase regardless of the different grain sizes, making the density contribution independent of the grain size with a form exponent $\varkappa = 0$.

Chemical development produces sponge-like silver conglomerates whose light absorption is proportional to the quantity of silver contained in a grain, which, in the extreme case, makes the form exponent $\varkappa = 1$. In the normal case one can expect a form exponent lying between $2/3 \leq \varkappa \leq 1$.

The integration of (18) may be performed with any grain size distribution one would like. An interesting approach to the solution is the representation by a convolution integral.

A distribution function consisting of a sum of exponential terms can be integrated particularly easily in an elementary way.

The only example we state here is the one-term exponential function according to (17), which yields

$$D = D_\infty \left[1 - \frac{1}{\left(1 + \dfrac{\eta}{\alpha} H\right)^{\varkappa+1}} \right]. \tag{19}$$

In the case of more summands in the distribution function we obtain as the solution a linear combination of terms like (19).

The sensitivity coefficient ε proves to be the ratio of the sensitivity density η and the grain-size distribution coeffizient α

$$\varepsilon = \frac{\eta}{\alpha} \tag{20}$$

and points to the strict correlation of sensitivity and grain size.

8. The influence of the radiative transfer

Passing through the opaque emulsion layer during exposure the light undergoes attenuation by absorption and scattering at the crystals of the silver halide grains. The correct attenuation function $\varphi(x)$ emerges as the solution of the integro-differential equation of the radiative transfer process.

A simple, but nevertheless sufficient solution is the exponential law of attenuation after LAMBERT

$$E_{\text{eff}} = E \exp\left(-\frac{\delta}{x_0} x\right), \tag{21}$$

x denoting the depth and x_0 the thickness of the layer, δ the optical density of the opaque emulsion coating, E the intensity and E_{eff} the effective intensity. The LAMBERT law holds better the more absorption predominantes over scattering within the layer.

We restrict the attenuation function $\varphi(x)$ to the one-term exponential function because of the difficulties arising in analytical integration in the case of using more complicated functions.

The entire layer may be divided into an infinite set of infinitesimal layers, whose densities according to the elementary blackening function are superposed, yielding by summation of all partial densities the integral over the depth of the layer

$$D = D_\infty \left\{1 - N \int\limits_0^{x_0} P[E\varphi(x), t] \, \mathrm{d}x\right\}, \tag{22}$$

N being a normalizing factor.

In the case of equal grain sizes the evaluation of the integral (22) turns out to be an exponential integral function, i.e. a relatively complicated function [6]. Paradoxically, taking account of the grain size in the form of a sum of exponential functions like (16) and (17) leads to a simplification of the analytical formulation of the blackening function, which can be expressed in terms of just standard functions. (We are confronted with the same effect as known for integral transformations — the FOURIER and the LAPLACE transformations.)

9. A simple but physically founded analytical characteristic function

The integration over the grain-size distribution and the thickness of the layer can be together represented by the following double integral:

$$D = D_\infty \left\{1 - N \int\limits_0^{x_0} \int\limits_0^{\infty} V^x \omega(V) \, P[\eta VE\varphi(x), t] \, \mathrm{d}V \, \mathrm{d}x\right\}. \tag{23}$$

The integration of (23) with the simplifying restrictions

1. SCHWARZSCHILD's blackening law is valid,
2. the sensitivity is proportional to the grain volume,
3. the grain-size distribution follows a one-term exponential law,
4. radiative transfer is given by LAMBERT's law,
5. there is physical development with a form exponent $\varkappa = 0$

yields the formula

$$\boxed{D = \frac{D_\infty}{n\delta} \ln \frac{1 + (\varepsilon H)^n}{1 + (\varepsilon H)^n \exp(-n\delta)}} \tag{24}$$

with $H = Et^p$ (the SCHWARZSCHILD product) as an independent variable.

This formula representing the characteristic curve holds very well in the region of the toe and satisfactorily over the quasi-linear middle part of the curve, but deviates in the

region of the shoulder. This is because of the simplifications listed above, which affect significantly only the shoulder of the curve.

The inversion of the formula (24) takes a form on as

$$H = \frac{1}{\varepsilon} \left[\frac{1 - \exp\left(n\delta \frac{D}{D_\infty}\right)}{\exp\left[n\delta\left(\frac{D}{D_\infty} - 1\right)\right] - 1} \right]^{\frac{1}{n}} \tag{25}$$

enabling thus the unique coordination of the D and H values by a set of two simple formulae.

The integration could be performed with fewer simplifications. The integrability of (23) is preserved if we use for the grain-size distribution as many terms as one likes; but only the form exponents $\varkappa = 0$ and $\varkappa = 1$ allow analytical integration [11]. Nevertheless, in every case a numerical integration could be performed, embracing all possible relations correctly. In all those cases the simplicity of a short analytical formula as well as the possibility of a direct inversion is lost.

10. The properties of the proposed blackening formula

The formula (24) contains only 4 parameters:

1. the saturation density D_∞,
2. the optical density of the opaque emulsion layer δ,
3. the step order n,
4. the sensitivity ε.

Therefore, only 4 values of the density spread over the characteristic curve are needed in order to determinate the whole function, being sufficient for interpolation. With more values an optimizing calculation could be carried out.

The characteristic curve drawn from the formula (24) is quite symmetrical to the point of inflection. This becomes obvious if we take the slope by differentiation. The derivative

$$\gamma = \frac{D_\infty}{\delta} \left[\frac{1}{1 + (\varepsilon H)^n \exp(-n\delta)} - \frac{1}{1 + (\varepsilon H)^n} \right] \tag{26}$$

consists of a difference of two LORENTZ-like terms with a coordinate shift between them. The gradient curve is bell-shaped and mirror-symmetrical in every case as shown in Fig. 1.

At the inflection point we have the gradient

$$\gamma_{ip} = \frac{D_\infty}{\delta} \tanh \frac{n\delta}{4}, \tag{27}$$

showing that the gradient can only increase up to a limiting value given by the ratio D_∞/δ, see Fig. 2.

The density at the inflection point is accurately

$$D_{ip} = \frac{D_\infty}{2} \tag{28}$$

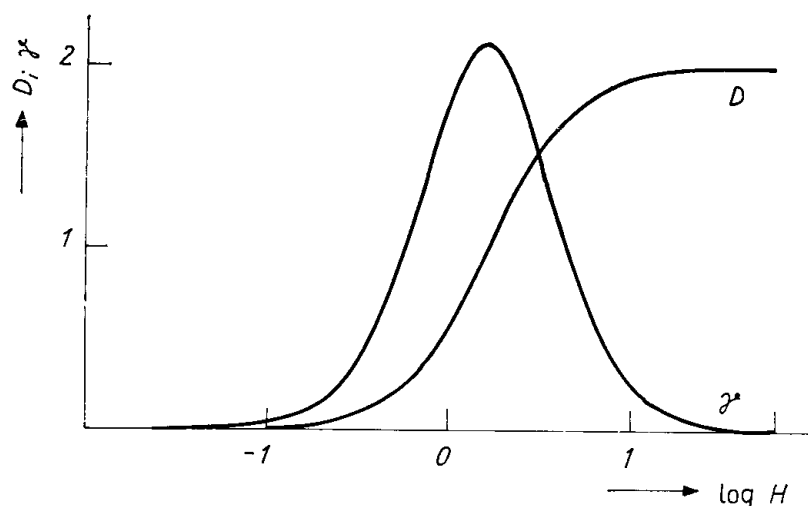in accordance with the torsion symmetry of the curve.

Fig. 1. Characteristic curve calculated using equation (24) and the related slope after equation (26), which exhibits a mirror-symmetrical bell-like shape. $D_\infty = 2$, $\delta = 1$, $n = 2$, $\varepsilon = 1$.
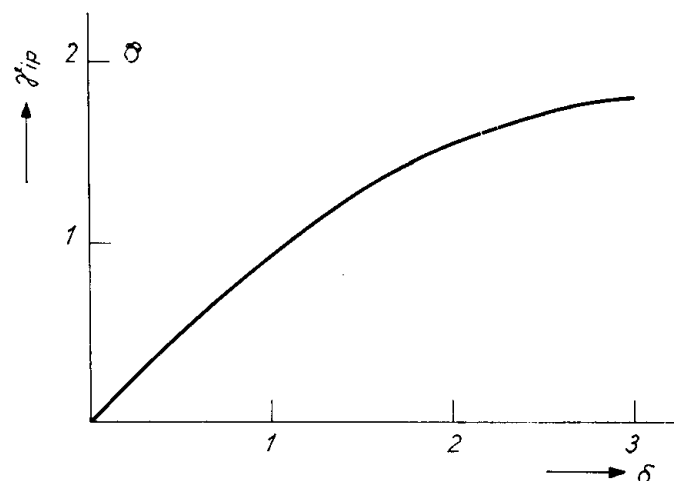
Fig. 2. The dependence of the gradient at the inflection point of the characteristic curve on the opacity density of the emulsion layer. $D_\infty = 2$, $n = 2$.

In order to demonstrate the influences of the parameters on the characteristic curve represented by (24) two families of curves are presented below. Figure 3 shows the effect of the step order n. For reasons of the kinetics theory of the step-like build-up of developing centers [5, 7] the step order is limited to $1 \leq n \leq 4$, n being near 1 for high-speed emulsions as used in astrophotography and being near 4 for unsensitized emulsions of high contrast as used for printing graphical purposes [14]. We can see that the contrast increases while the sensitivity decreases with growing values of n. Varying n the shape of the curve is either contracted or expanded, respectively, in direction of the abscissa, whereas the inflection point preserves its position at half the saturation density. This is because of the scale-determining effect of the exponent n in the power function H^n on the logarithmic scale.

The exponent n has been used by de VAUCOULEURS [20] to represent the toe of the characteristic curve by a power function.

Fig. 4 demonstrates the effect of the optical density δ of the opaque emulsion layer. Because of the increasing absorption in the layer during the exposure the curve is stretched to a nearly linear middle part but also flattened with extension of the exposure latitude and, of course, shifted to the less sensitive side. This effect may be produced on a given photomaterial by dyes acting during the exposure. The dyes may be washed out after the development, thus supplying a better transparency of the finally resulting photograph. Such a method is recommended for correct reproduction in the printing process.

The sensitivity parameter ε causes a shift along the log H-axis; likewise the saturation density D_∞ stretches the ordinate. Both influences on the characteristic curve need not be demonstrated explicitly. It should, moreover, be considered whether the coefficient ε could be used as a measure of the sensitivity because it is physically founded and independent of particular practical references.
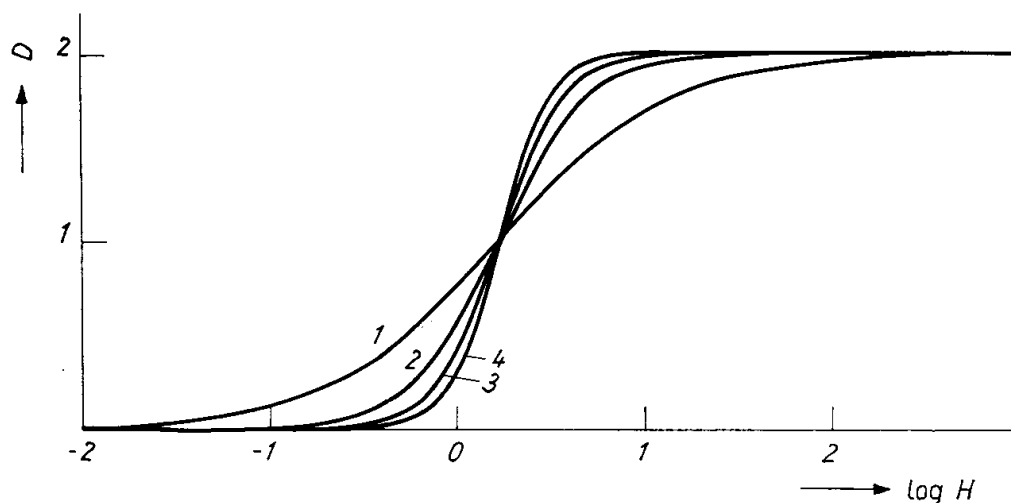

Fig. 3. Family of characteristic curves by the parameter n. $n = 1, 2, 3, 4$. $D_\infty = 2$, $\delta = 1$, $\varepsilon = 1$.

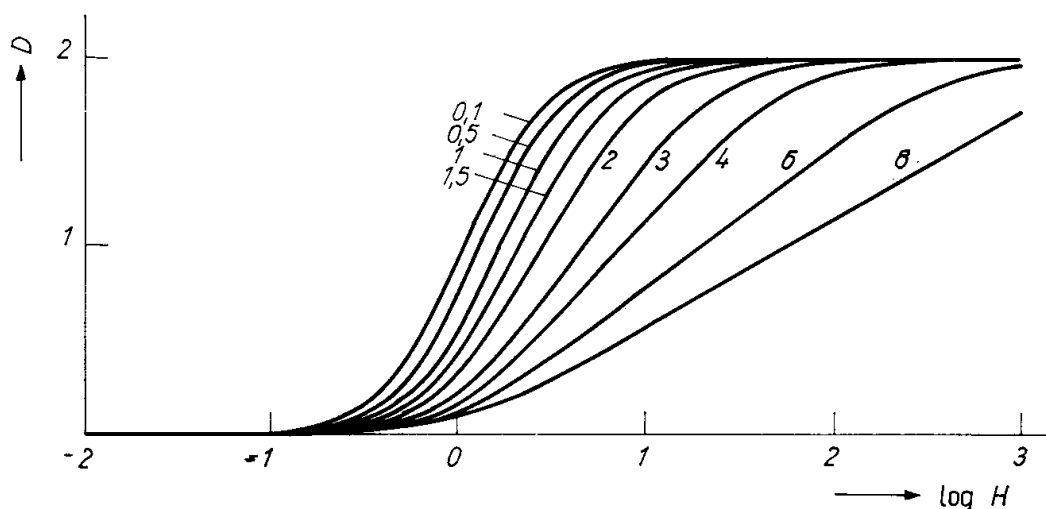
Fig. 4. Family of characteristic curves by the parameter δ (D versus log H). $\delta = 0.1$, 0.5 1, 1.5, 2, 3, 4, 6, 8, $D_\infty = 2, n = 2, \varepsilon = 1$.

11. Significance and determination of the four "blackening parameters"

The four parameters D_∞, δ, n, and ε represent physically founded "characteristic quantities" of the blackening function (therefore called "blackening parameters"), which are valid also for more complicated versions of blackening formulae — may be either analytical or empirical ones. In the 4-parameter formula (24) these magnitudes attain their clearest manifestation. They may serve even for definitions as, e. g., the sensitivity, the steepness, the linearity, the exposure latitude, the density modulation, the saturation density. The 4 "blackening parameters" maintain their physical significance also in optimization procedures applied to sensitometric measuring data as shown by ESCH-RICH [3], which lead to a simultaneous determination of the parameters as the whole set. Moreover, three of these parameters, namely D_∞, δ, and n, may be determined seperately for their own; the fourth parameter ε depends to some degree on the others.
The measuring approach for the 4 parameters is:

1. D_∞: The saturation density is to be measured on a totally overexposed and then developed photographic material (excluding solarization and other photographic effects that the formula (24) is not valid for).

2. δ: The optical density of the opaque emulsion layer is to be measured on the undeveloped emulsion layer of the photographic material. (The densitometer light should be of the same spectral composition as that one used to expose the photographic material to be investigated, but using minimal light quantities to avoid photolytic blackening during the measurement.)

3. n: The development step order is to be determined by transforming the ordinate (D-axis) of the characteristic curve into a logarithmic scale. The toe of the curve is formed by this way to a straight line, the gradient being the parameter n [14]. This is because the limiting case of formula (24) for diminishing light quantity H emerges to a power function

$$D \approx D_\infty(\varepsilon H)^n \, \frac{1 - \exp(-n\delta)}{n\delta},$$ (29)

the exponent n of which may be represented in double logarithmic coordinates as the gradient.

4. ε: The sensitivity is derived from the characteristic curve by determining the parallel shift of the curve along the log H-axis taking into account the knowledge of the other parameters determined already in advance, for ε is defined as the sensitivity of the individual grains regardless of the radiative transfer taking place in the emulsion layer during the exposure. Thus, in the region of the toe ε may be derived for a definite pair of values H, D using equation (29), which would be quite in accordance with the usual definition of the sensitivity as the light quantity at a definite low density level of the characteristic curve.

Another approach is given by using the most characteristic point of the characteristic curve: the inflection point. Then we have with the light quantity H_{ip}, inserting equation (28) into (25),

$$\varepsilon = \frac{1}{H_{ip}} \, \frac{1 - \exp\left(\dfrac{n\delta}{2}\right)}{\exp\left(-\dfrac{n\delta}{2}\right) - 1},$$ (30)

which brings about that this formula does not contain the saturation density D_∞. Since the opacity density δ decreases, the fraction containing n and δ tends to unity. In the limiting case, which corresponds to full transparency of the emulsion layer as being valid for exposure on X-rays, the sensitivity parameter ε represents the reciprocal light quantity at the inflection point, from which it is shifted off only by action of the opacity density of the emulsion layer.

12. Conclusion

Allowing for special approximations of the complicated conditions of the photographic process, a simple blackening formula related to 4 parameters can be derived, which proves to be appropriate for different analytical treatments requiring a mathematical formulation of the characteristic curve.

The derivation performed in this way makes it evident that the monotonical functional relation between exposure and density increasing from zero to the saturation level as well as the sigmoid shape of the characteristic curve are caused in full already by the statistics of size and location of the silver halide grains in the emulsion layer regardless of the process of speck build-up within the grains — as we can see when we replace the term $(\varepsilon H)^n$ in equation (24) by the average number of centers per grain $X = \bar{z}$ according to equation (6), and defining

$$T = \exp(-n\delta) \tag{31}$$

as the effective total transparency of the opaque undeveloped emulsion layer, resulting from the radiative transfer through the layer. We obtain then the more generalized formula

$$D = D_\infty \frac{1}{\ln(1/T)} \ln \frac{1 + X}{1 + XT}, \tag{32}$$

which reflects only the functional relation between the average occupation number of development specks and the density, using as parameters the saturation density D_∞ and the effective transparency T. Equation (32) can be regarded as normalized with respect to n and ε. The quantity X may be an arbitrary function (even a not monotonical one as in the case of solarization) that summarizes the result of the photographic process taking place in the crystalline lattice of the silver halide grain, which can be described — like before — as a function of the Schwarzschild product Et^p or — with a wider extension of validity — by equation (7) but done completely only by the full application of the reaction kinetics theory [7, 10]. So, in the mathematical formulation of the characteristic curve the photographic process and the shaping of the curve, which otherwise is entangled awfully by complicated integrations, by this way will be (approximately) disentangled.

Furthermore, the 4-parameter formula is of practical use, too. Thus, the set of formulae joining the functionally related quantities H and D may be used to compute the characteristic curves and the information transfer of a manifold copying process, rendering even the possibility of control and computer-regulated optimization. The simplicity of the formula (24) including its inverse version (25), makes it well-suited to application in computer-programs, especially when a subroutine of such a formula is run frequently as, e.g., for the recalculation of the intensity from densitometer tracks.

The formula (24) represents a basic version of a physically founded formulation of the characteristic curve. The course of its derivation is open for any further generalization.

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