Analytical treatment and numerical calculation of conversion series of radionuclides by means of matrix functions

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Abstract

The analytical treatment of a process of radioactive reactions accompanied with conversion of chemical elements (spontaneous decay or build-up of the nucleus by capture of neutrons) can be afforded advantageously by formulating the system of linear differential equations by matrices, which yields at once an exact solution in its full entirety.

The solution is given by a matrix transformation of the nuclide components arranged as a vector of the nuclides, which is transformed in a "reaction space" by a resolving matrix during a transition process from an initial composition to the end composition. The resolving matrix is represented as an expansion of a matrix exponential function, yielding immediately the algorithm for the numerical calculation in a computer program.

The linearity of the system of differential equations offers also the solution by using the Laplace-transformation, which is discussed concerning advantages and disadvantages compared to the expansion of the matrix exponential function (Neumann's row).

The application of vector algebra gives some interesting insight into the interconnection of the reacting components of the system: Thus, successive processes with different transition coefficients are commonly not commutative because of the noncommutativity of matrix-multiplication. The measured total radioactivity of the element composition is the scalar product of the column vector of nuclides and the row vector of detection probability. Every selected transition system is represented as a submatrix of the "global reaction matrix" comprising all possible nuclear transitions from hydrogen to transuranium.

Some numerical examples are given for the calculation of the uranium row and the conversion of gold to mercury.

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