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Analytical representation of the photographic characteristic curve by matrix functions [1]

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Abstract

For the derivation of an analytical formula of the relation between the exposure and the blackening of the photographic layer it is assumed that the build-up (and decay) of development specks can be treated as a **multi-step reaction chain** with stochastically conditioned transition probabilities from step to step. Hereby the forward reactions are determined by the free electrons released in the crystal lattice by action of the photoelectric effect, whereas the back reactions are attributed to the thermal and chemical dissociation as well as to the direct influence of the photoelectric effect on the already created specks.

The initial concentration of specks in all steps of the reaction chain is formulated as a vector $\mathbf{c}(0)$ (column matrix) in a multidimensional reaction space, which is to be transformed during the exposure into a resulting vector $\mathbf{c}(E,t)$ by the transformation matrix $\mathbf{B}(E,t)$.

$$\mathbf{c}(E,t) = \mathbf{B}(E,t)\ \mathbf{c}(0). \tag{1}$$

The matrix $\mathbf{B}(E,t)$ is called here the **"exposure matrix"** because of its dependence on the parameters of the exposure, the light intensity E and the time t.

Multiple exposures are expressed in matrix writing style as multiple transformations, whereby the resulting transformation matrix proves to be the product of the transformation matrices of individual exposures.

Especially important for the analytical description of the photographic **double-exposure effects** is the **non-commutativity** of matrix multiplication.

Accounting for the row vector of the development probabilities $\overline{\mathbf{w}}$, the grain-size distribution $\omega(a)$ of the sensitive volume V, and the depth x down to the thickness x_0 of the layer, the analytical formulation of the characteristic blackening curve is given by the double integral

$$S(E,t) = \frac{S_0}{\overline{a^2} x_0} \int_0^{x_0} \int_0^{\infty} a^2 \omega(a) (1 - e^{-V \overline{\mathbf{w}} \mathbf{B}(E,t) \mathbf{c}(0)})\ \mathrm{d}a\ \mathrm{d}x. \tag{2}$$

The numerical evaluation of the blackening function is performed by computer programs, which allow the calculation of the exposure matrix as an expansion of a matrix exponential function. The calculation yields concordance with the exposure-blackening behavior of real photographic layers – thus among others also in case of several **exposure effects** (SCHWARZSCHILD-effect, WEINLAND-effect, VILLARD-effect, HERSCHEL-effect, ultra-short exposure effect, intermittence-effect).

Comment of the author in 2008:
The manuscript of the article was submitted to the Journal *Annalen der Physik* on February 23rd, 1971 in form of a *Short Announcement*, in order to secure the priority especially for the matrix formulation of the kinetic process of the step-like build-up of development specks at the silver halide grains in the photographic emulsion. The whole theory was presented to public two months later in the habilitation thesis of the author, which was defended at the *Technische Universität Dresden* on April 26th, 1971.
The theses are available in German by `www.ewald-gerth.de/40thesen.pdf`.

[1] Article in German available by `www.ewald-gerth.de/36.pdf`