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Analytical representation of the photographic characteristic curve accounting for the Schwarzschild-effect [1]

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Abstract

The build-up process of development specks at crystals of silver bromide in a photographic emulsion is regarded as a chain of equilibrium reactions, which are characterized in that the forward reactions are determined by the concentration of free electrons released in the crystal during the exposure to light, whereas the back reactions take place due to thermal decay and the photoelectric effect acting directly onto the already created specks. The specks grow from step to step by recharging with free electrons. At low intensity, the saturation concentration of electrons is proportional to the intensity of the exposure, but in the case of high intensity the electron concentration is proportional to the square root of the intensity.

On the assumption that specks of the first degree are very unstable, a reaction equilibrium occurs already in the first reaction step, resulting in the reduction of the order of the exposure time by one degree. Accounting for the transition probability, SCHWARZSCHILD's well-known law of blackening

$Et^p = \text{const},\quad (E - \text{intensity},\ t - \text{time},\ p - \text{SCHWARZSCHILD-exponent})$

can be derived already on plausible grounds.
The formula describes only the long-term exposure effect. The validity of the blackening law is extended to long and short exposure times with a transition region expressed by the formula

$(\sqrt{1+\varepsilon E}-1)t^p = \text{const}.\quad (\varepsilon - \text{sensitivity coefficient})$

The analytical representation of the characteristic curve is based on the POISSON probability of the silver bromide crystals reduced by the developer to metallic silver with the average number of cell occupation as a function of the SCHWARZSCHILD-product Et^p (for low intensity E and long time t only) or for the entire diapason of low and high intensity together (long and short times, resp.),

$$S = S_o\Big[1 - \frac{1}{x_o}\int_0^{x_o} \mathrm{e}^{-\left((\sqrt{1+\varepsilon E}-1)t^p\cdot \mathrm{e}^{-Dx}\right)^n}\mathrm{d}x\Big].$$

(S – blackening, x – thickness of the layer, D – optical density of the undeveloped layer)

The formula holds very well in the range from the toe up to the quasi-linear part of the characteristic curve but shows increasing deviation at the shoulder the more saturation of the density is reached. This is because the grain size distribution is not accounted for, which was introduced in later derivations of the characteristic curve: www.ewald-gerth.de/36abs.pdf

Comment of the author in 2008:
The manuscript of the article was submitted to the journal *Zeitschrift für wissenschaftliche Photographie, Photophysik und Photochemie* (quoted: Z. wiss. Phot. **59** (1965), 1) on August 8^{th}, 1964 in order to secure the priority for the explanation of the SCHWARZSCHILD-effect as an outcome of the kinetic process of the step-like build-up of development specks at the silver halide grains in the photographic emulsion. The article is an excerpt published in advance of the author's thesis on double exposure effects, which was defended at the *Pädagogische Hochschule Potsdam* on November 19^{th}, 1965.
The theses of the thesis are available in German by www.ewald-gerth.de/19thesen.pdf .

[1] Article: www.ewald-gerth.de/16.pdf On Schwarzschild-effect: www.ewald-gerth.de/22abs.htm