Ten years magnetic modelling of stars by field sources - a review

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Motivation and beginning

Half a century ago - since Babcock's epoch-making work -
an enormous quantity of **observational data**
on magnetic stars has been compiled,
waiting for **analysis and interpretation**.

Four decades from then it was tried to derive the **structure**
of the magnetic surface field by inverse **reduction procedures**.

One decade ago Glagolevskij and Gerth proposed
a **modelling method** to construct the magnetic field
out of its sources by **straightforward calculation**.

History:

September 1994 – after a proposal of Yu.V. **Glagolevskij** – ***Start*** of the program ***Stellar Magnetic Modelling***:

1995 First poster representation in **Vienna** at IAU Symposium 176: **www.ewald-gerth.de/90pos.pdf**

1997 First publication in the **Proceedings** of the Workshop in Vienna: **www.ewald-gerth.de/90.pdf**

Axiomatic statements and construction of a spatial vector field

A vector field fills the space

1. starting from a **source** and continuously by a directed stream, the field lines ending in a hollow (negative source)
2. or circulating around a **directed axis**.

After a theorem of the potential theory –
every vector field is determined by linear superposition of the fields of

1. sources

2. and **vortices**.

Of special interest for astrophysics are:

1. **velocity** vector fields of mowing material,
2. **electric** vector fields outgoing from electrically charged particles,
3. **magnetic** fields surrounding the moving electrically charged particles.

Reference: **www.ewald-gerth.de/111.pdf** **www.ewald-gerth.de/111pos.pdf**

Analogy of electric and magnetic fields

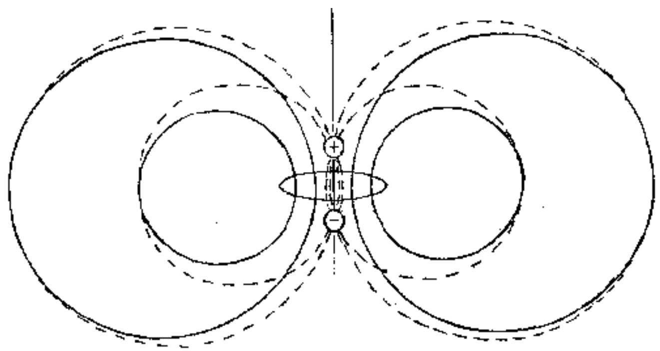
Electric and magnetic fields are closely bound together by **Maxwell**'s laws which enables the **propagation of waves** – the **dynamic case** of fields .

The topographic field structure of the star is described only by the **stationary case** – as given naturally by electrical fields with electrical charges and analogously for magnetic fields by ***virtual magnetic charges***.

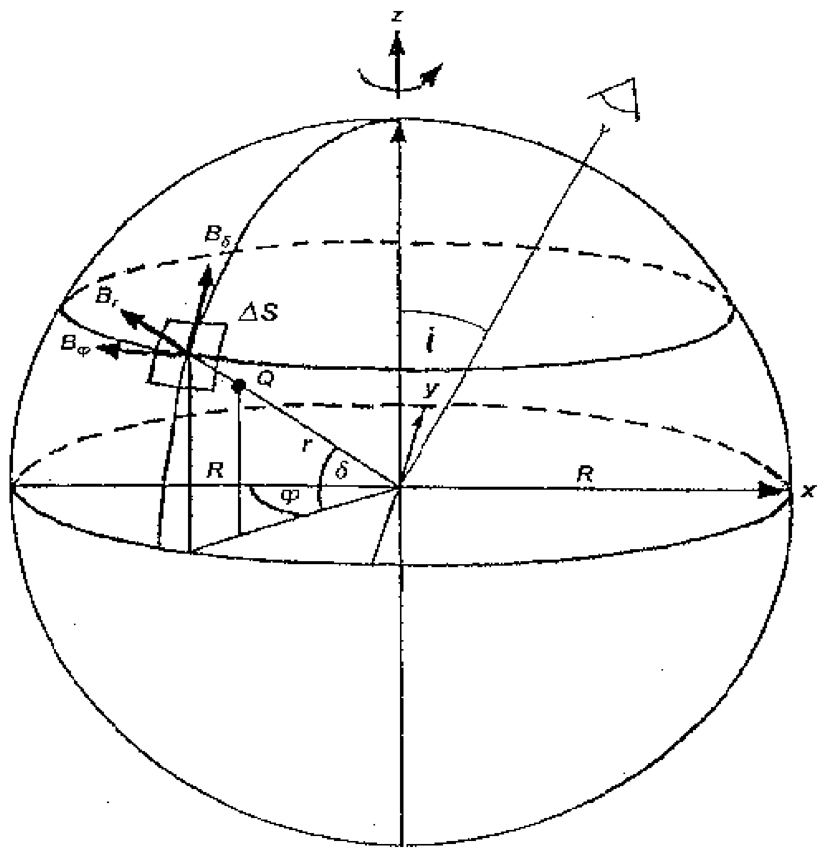
Magnetic dipoles can be constructed by

1. **ring-like aligned vortices** representing the circulating electrical current,
2. **a pair of "virtual magnetic charges"** of opposite polarity.

Both of them are ***identical*** *by shrinking* of the ring or the distance between the charges infinitesimally to zero.

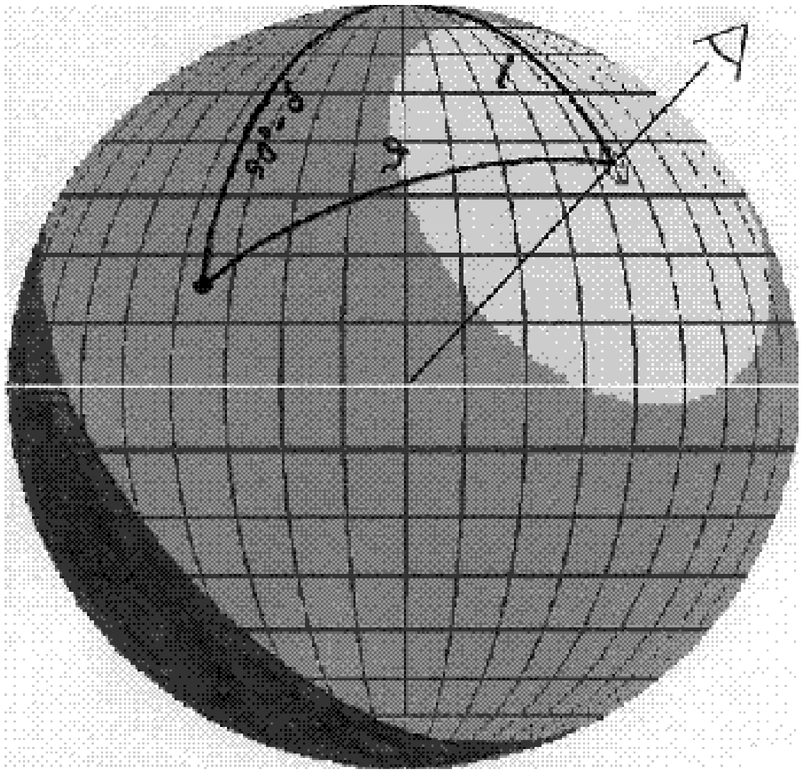


Coordination of a monopole



1. Projection of the field outgoing from a source *Q* onto a sphere element Δ*S*
2. Axis of rotation – perpendicular
3. Line of sight – tilted by inclination angle *i*
4. Distance of *Q* from the center – radius *r* < > *R* (inside and outside)

Observational aspect window of the magnetic surface field



All detectable physical magnitudes on the surface – including the magnetic field – are viewed through the aspect window.

The projection of the sphere on the line of sight : $\cos \upsilon = \sin \delta \cos i + \cos \delta \sin i \cos \varphi$

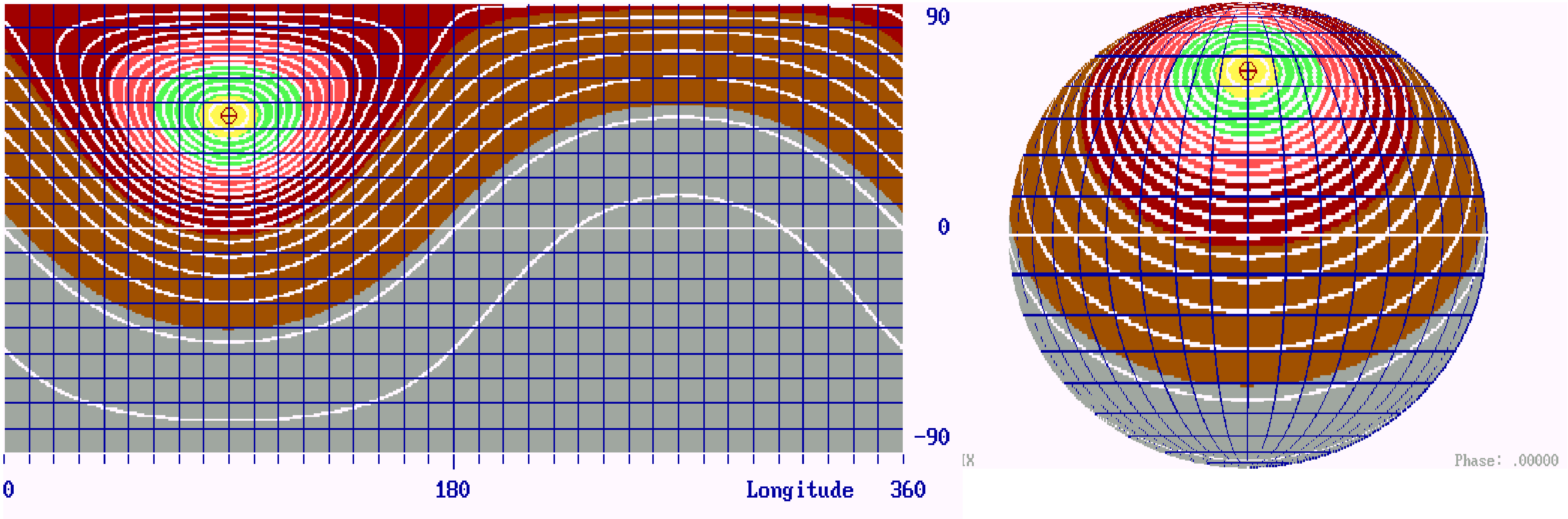
The opposite side to the viewer (black) – zero. **Around the line of sight (gray) – limb darkening .**

The aspect varies by rotation of the star – modulating the magnetic field.

The phase curve of the effective magnetic field is the result of convolution of the surface distribution with the aspect window.

Reference: www.ewald-gerth.de/119.pdf

Topographic coordination of a monopole field source in a *Mercator map* by a rectangular matrix

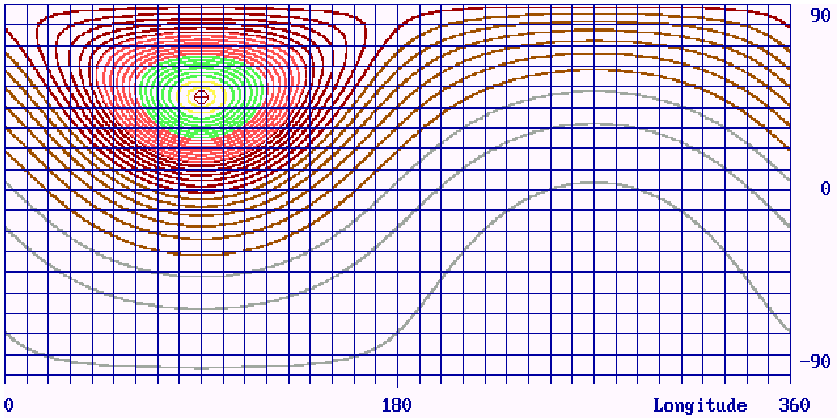


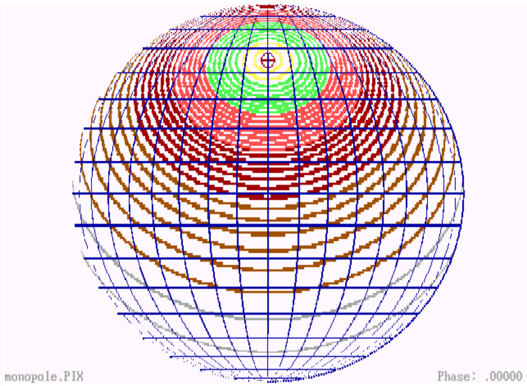
The **monopole** distinguishes itself as ***the* elementary field source** –
the “brick” of the building of any complex field configuration.

The mathematical representation of the coordinated field of a **point-like source** is comprised in a standard function – to be used for a **standard algorithm**.

Reference: www.ewald-gerth.de/111.pdf www.ewald-gerth.de/119.pdf

Comparing the map of a vortex to that of a monopole



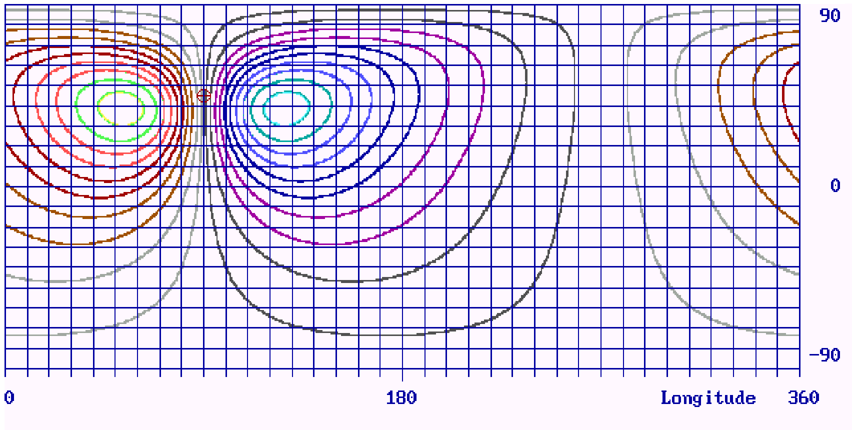


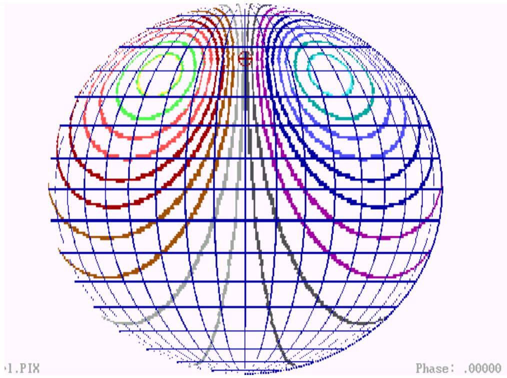
Monopole

$$\text{grad } U = \frac{\partial U}{\partial x}\mathbf{i} + \frac{\partial U}{\partial y}\mathbf{j} + \frac{\partial U}{\partial z}\mathbf{k}$$

The conservative vector field is defined as the gradient of the potential.

Ref.: www.ewald-gerth.de/105.pdf
www.ewald-gerth.de/105pos.pdf





Vortex

$$\text{curl } \boldsymbol{I} = \left(\frac{\partial I_z}{\partial y} \mathsf{B} \frac{\partial I_y}{\partial z}\right)\mathbf{i} \ \mathsf{A} \ \left(\frac{\partial I_x}{\partial z} \mathsf{B} \frac{\partial I_z}{\partial x}\right)\mathbf{j} \ \mathsf{A} \ \left(\frac{\partial I_y}{\partial x} \mathsf{B} \frac{\partial I_x}{\partial y}\right)\mathbf{k}$$

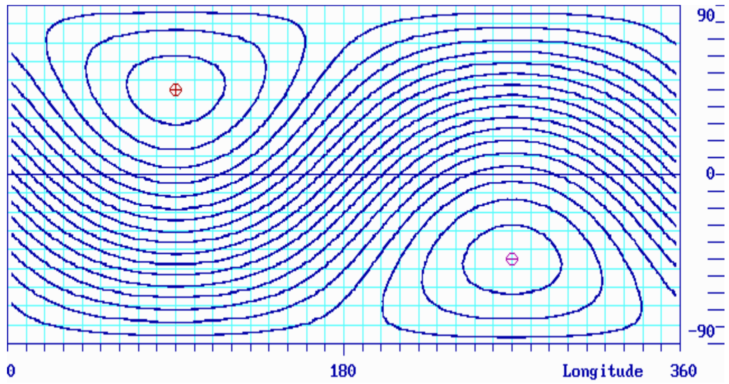
The vortex vector field is defined as the curl of the streaming current.
x, y, z coordinates; **i**, **j**, **k** Cartesian unit vectors;
U potential, $\boldsymbol{I}$ electrical current components I_x, I_y, I_z

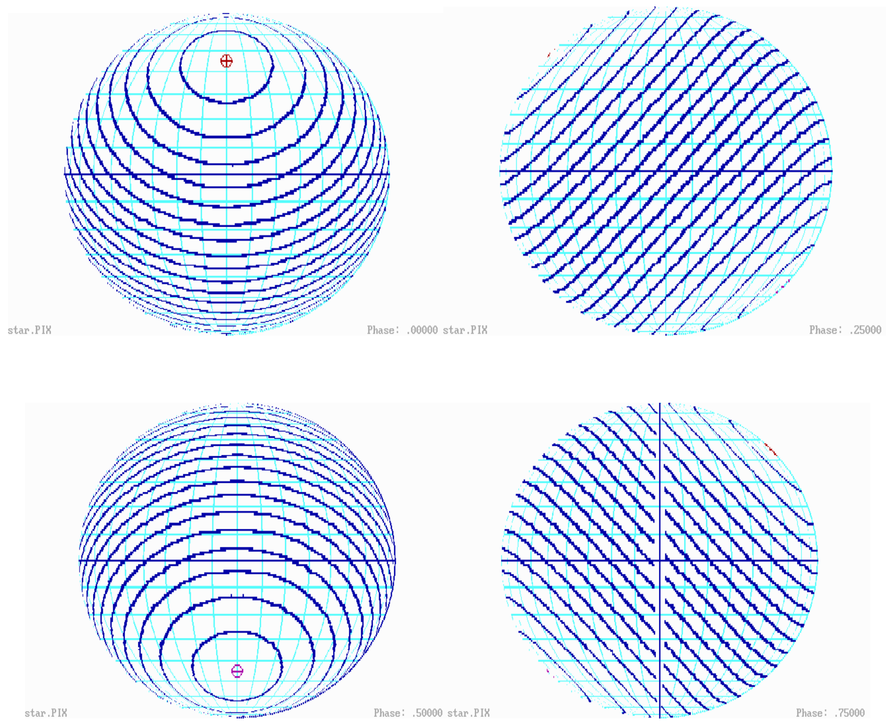
Parameters: $r = 0.5\ R$ $\varphi = 90°$ $\delta = 45°$

The **linearity** of the differential operators enables the **superposition** of a multiplicity of singular fields of sources or/and vortices in a successive arithmetic procedure.

Despite the **equivalence** of both of them, the point-like source of a potential field proves to be much more **convenient for programming and efficient computing**.

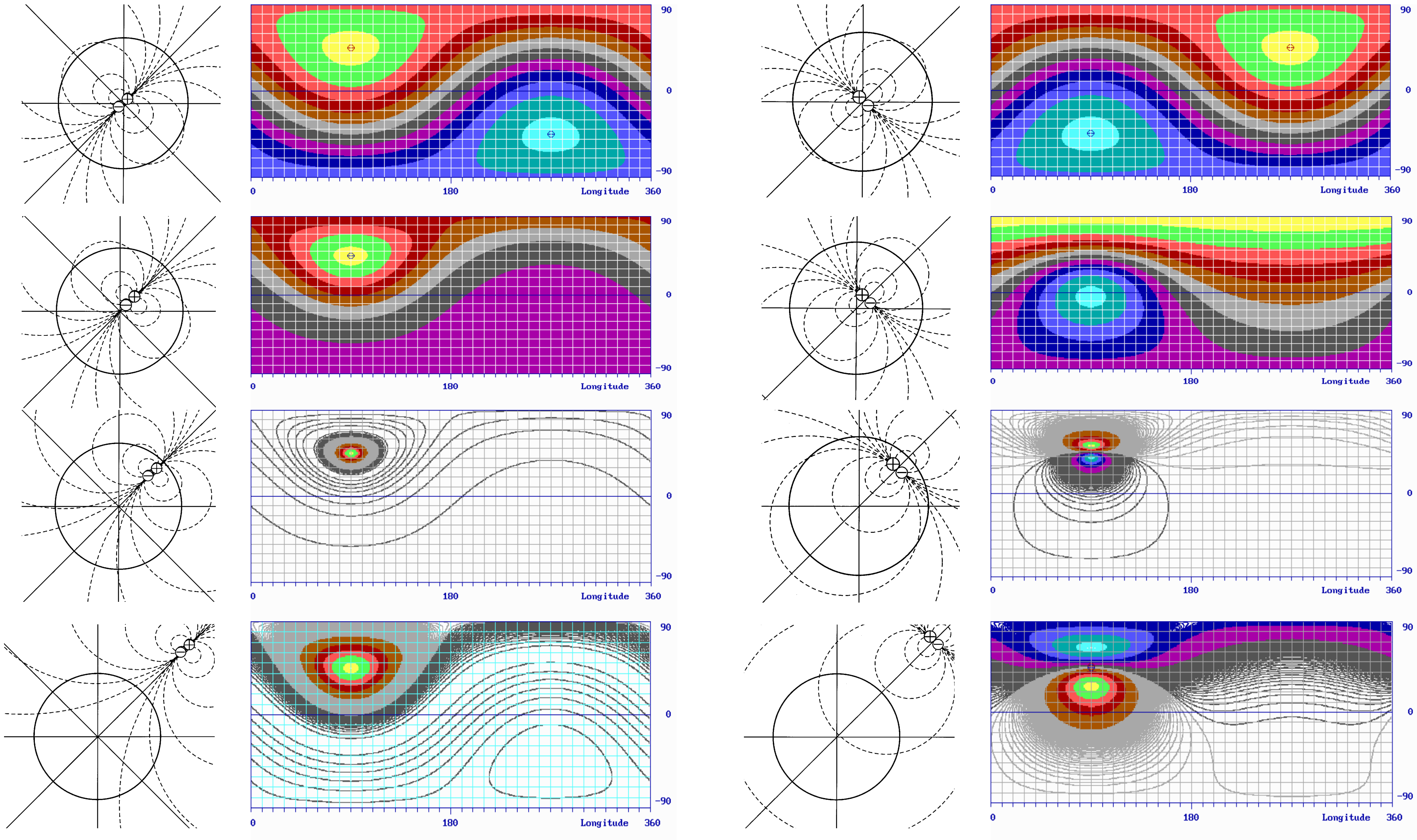
The field of a dipole consisting of a pair of oppositely charged sources





Mercator map with globes to the phases 0.25, 0.5, 0.75, and 1.00, arranged as a matrix.

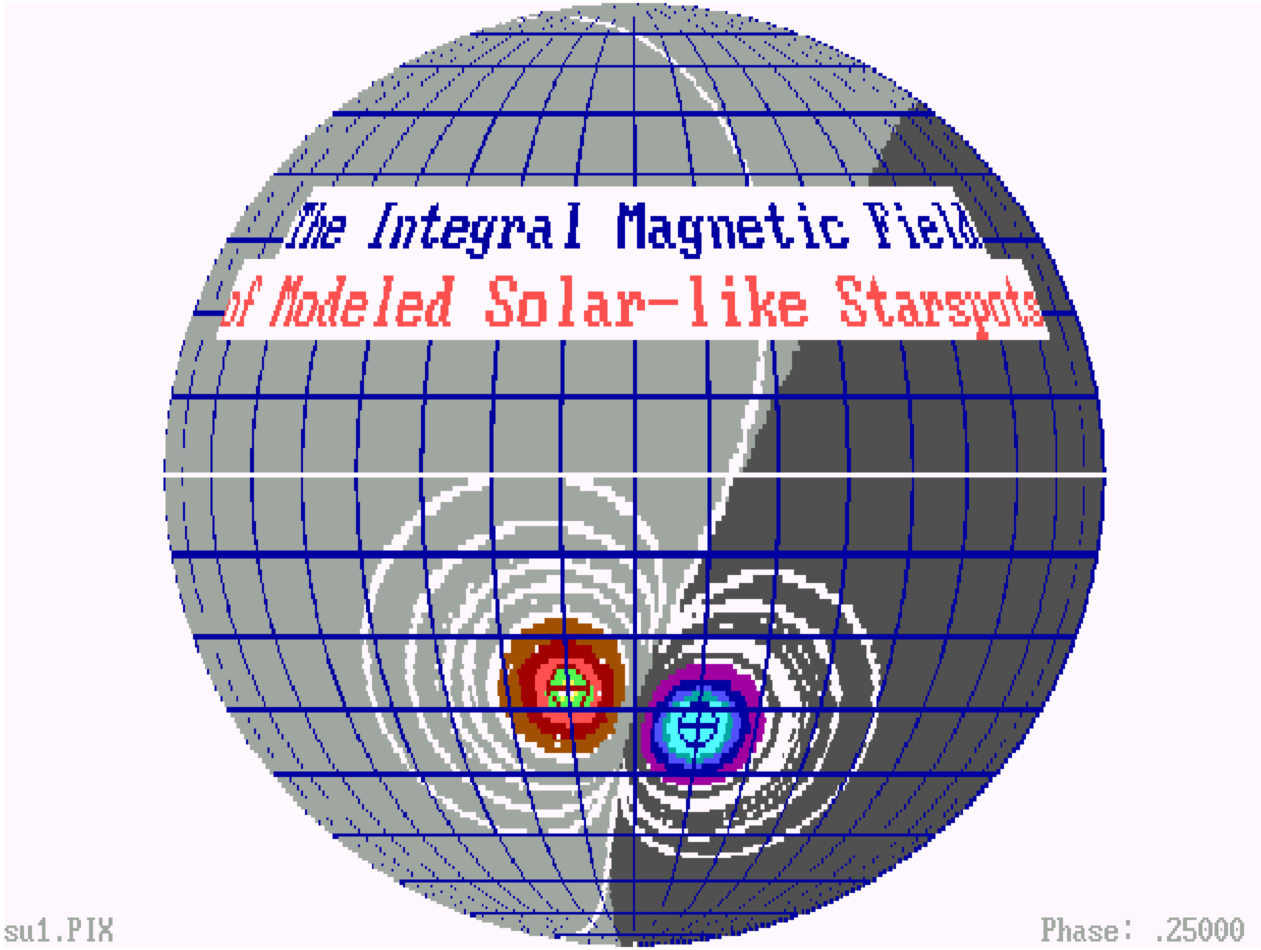
The decentered dipole – inside and outside the star



Magnetic dipole seen at $i = 45°$, shifted from the center outward. Left: alignedly to the line of sight. Right: perpendicularly to the line of sight.

Reference: www.ewald-gerth.de/117.pdf www.ewald-gerth.de/117pos.pdf

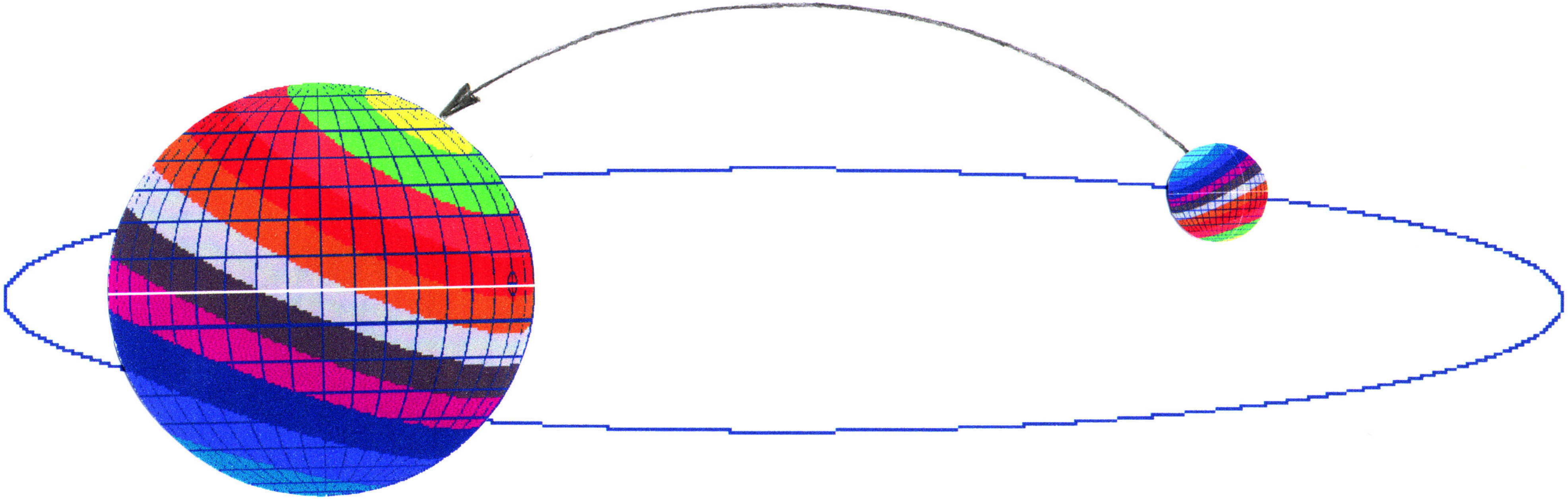
.A solar-like starspot – an example of an extremely decentered magnetic dipole



The magnetic moment is arranged shallowly under the surface.
The starspot can be represented either by

1. a pair of oppositely charged sources www.ewald-gerth.de/102.pdf
2. or by a vortex. www.ewald-gerth.de/103pos.pdf

An external magnet – an outmost decentered dipole

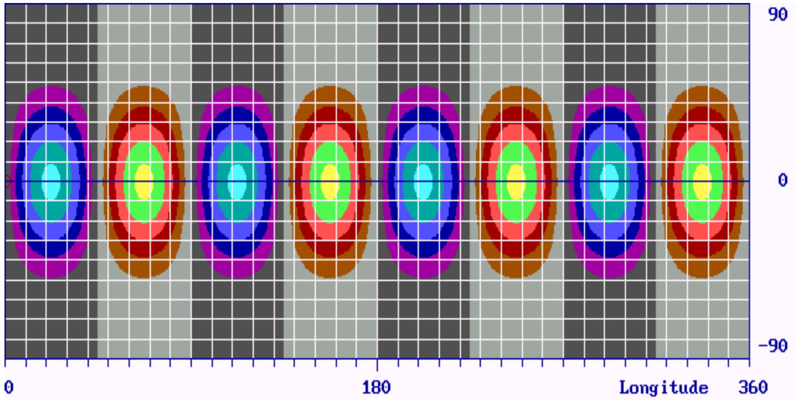


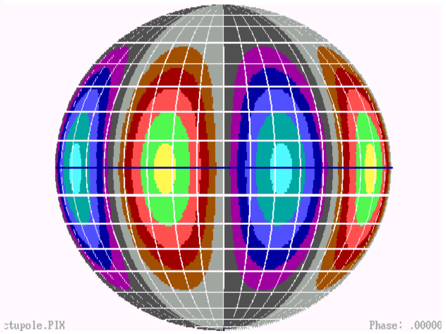
The **external magnet** as a companion of a **binary system** revolves around the primary star on an orbit, which determines the varying distance and radial velocity.

A **candidate** for such a system with external magnetic influence seems to be the **supergiant star *ν*Cep**, the unexpected strong magnetic field of which (+2000 Gauss) was detected in 1978 by **Gerhard Scholz** using the 2m telescope in Tautenburg.

Ref.: www.ewald-gerth.de/53.pdf www.ewald-gerth.de/54.pdf www.ewald-gerth.de/63abs.htm www.ewald-gerth.de/105.pdf

Description of the magnetic surface field by spherical harmonics





Demonstration of an octupole

a) Equatorial belt of 8 poles
Only coefficient: $A_4^{-1} = 1$, all other $A_n^m = 0$

Phase of the globe: $p = 0.25$, corresponding to $\varphi = 90°$

Reference: www.ewald-gerth.de/111.pdf
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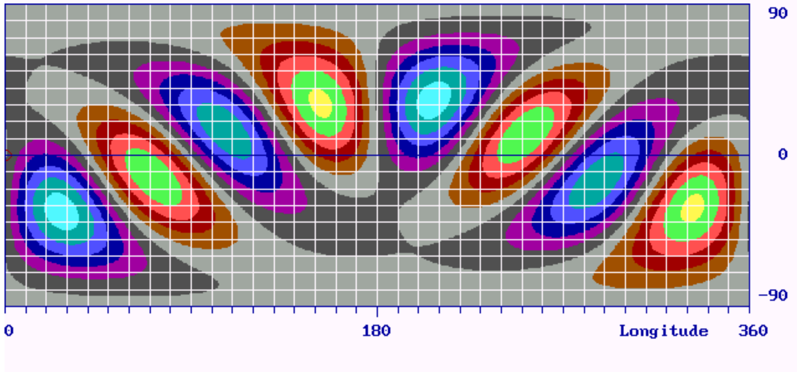
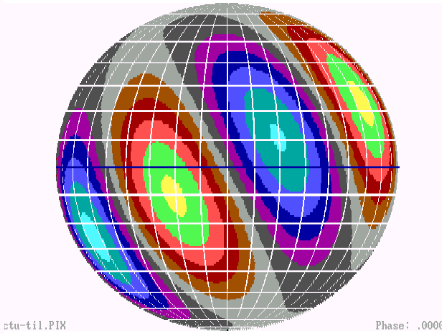




b) Belt of 8 poles, tilted by 40°

9 Legendre coefficients of the fourth degree:
$A_4^{-4} = 1$, $A_4^{-3} = 3$, $A_4^{-2} = 6$, $A_4^{-1} = 9$, $A_4^{0} = 1$,
$A_4^{1} = 0$, $A_4^{2} = 0$, $A_4^{3} = 0$, $A_4^{4} = 0$

Phase of the globe: $p = 0.25$, corresponding to $\varphi = 90°$

Can magnetic stars be modelled by spherical harmonics ?

Spherical harmonics describe an analytical function relating to the sphere plane as diapason of definition.

The function is analytically represented by an expansion of Legendre's associated spherical polonomials.

The coefficients of the Legendre-expansion serve for the analytical description – without physical meaning.

The description of the stellar magnetic surface field is valid only for the star's surface – neither inside nor outside.

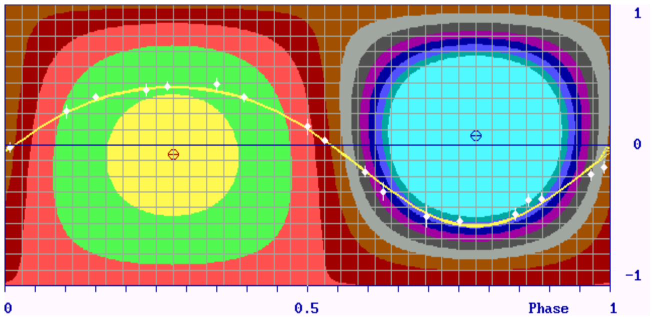
Spherical harmonics are not appropriate for magnetic modelling!

The covering of the magnetic surface field by inhomogeneous distribution of chemical elements

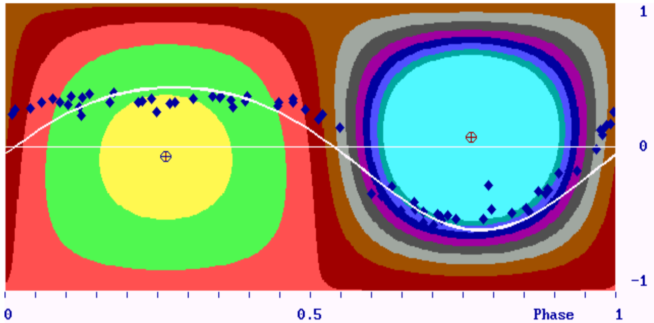
The observed magnetic field is an **integrated** one, which exists independently of visibility and detection.
The integration is related to the information transferring medium: the **spectral line profile**.
The **element distribution** acts like a transparency filter for the field.

Exempli gratia: Ring-like element distribution around the magnetic poles at the CP star 53 Cam

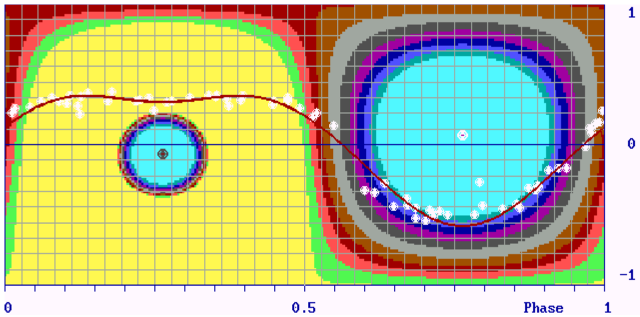
The observational data are fitted to the computed phase curves and coordinated to the maps.



Photoelectric observations of Borra and Landstreet (1977)



Photographic observations of Babcock (1960), plotted to the map.



Overlay of a ring-like filter to the map. Fitting to Babcock's data.

The element distribution disturbs decisively the observation of the magnetic surface structure.
Different covering of the magnetic poles mimics even the existence of unipolar magnetic fields.
An a priori unknown element distribution makes any inverse reconstruction of the field hopeless.

Reference: www.ewald-gerth.de/117pos.pdf

Conclusions

The magnetic field emerges from **sources** and **vortices**.

Any complex magnetic field is the result of linear **superposition** of **elementary fields**.

The magnetic field **fills** the **entire space** and penetrates spatial planes within the space from any side.

The distribution of the magnetic field on the surface of a star is represented as a **rectangular matrix** of the surface **elements**.

The **standard algorithm** of the computation of stellar surface fields relates to the **elementary sources**.