The effect of surface distribution of elements on the integral magnetic field of a star

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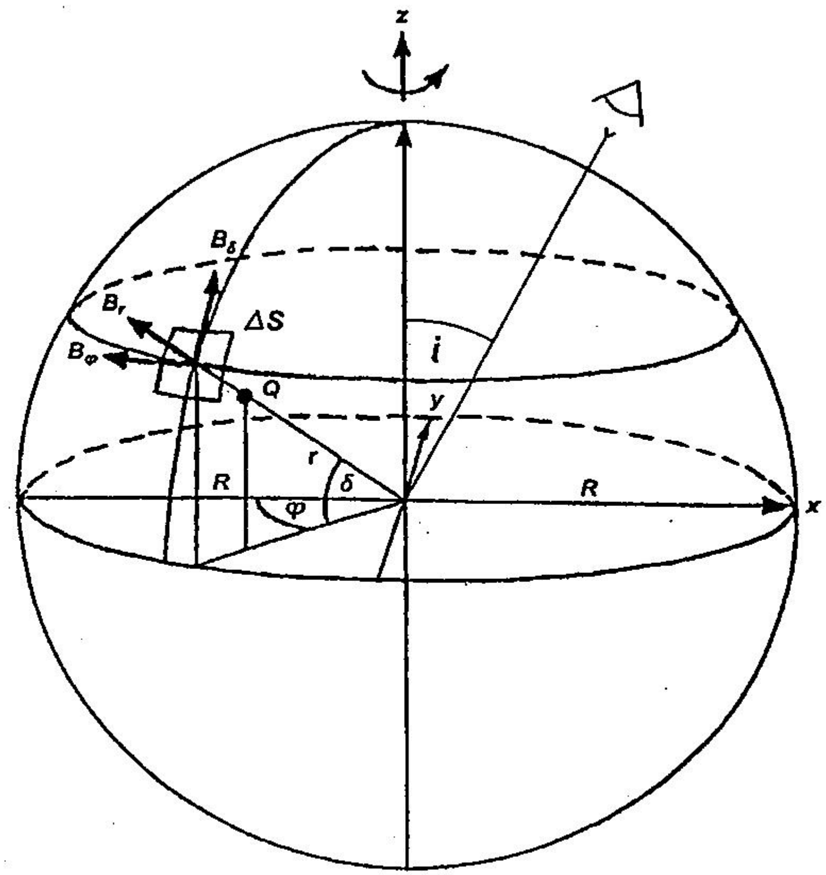
Statements:

1. The magnetic field of a star is the result of integration over its visible hemisphere, related to the information transferring medium: the **spectral line profile**.

2. Modelling the field structure has to account for the **element distribution** in the star's atmosphere.

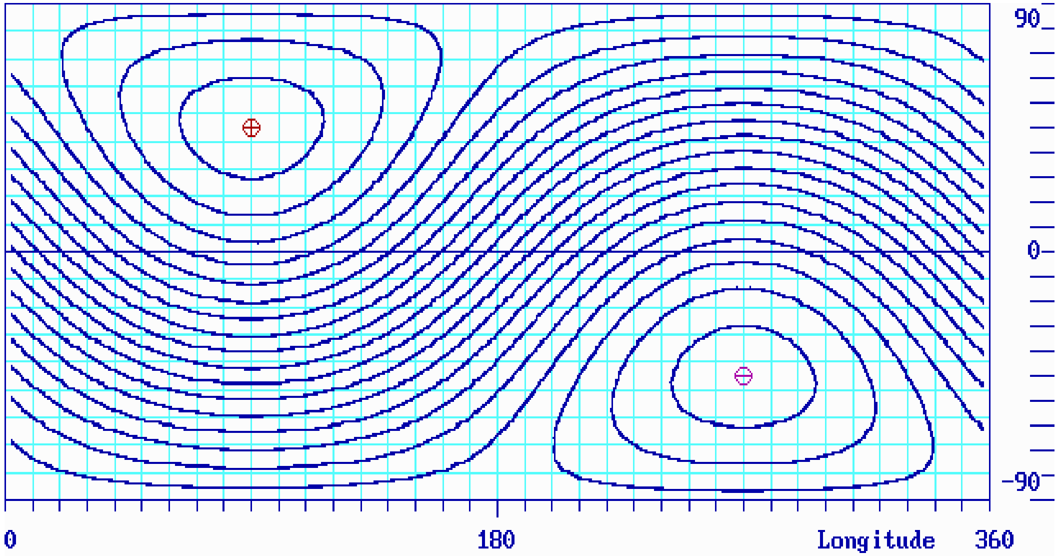
3. The theoretically predicted **ring-like arrangement** of elements around the poles affords the best agreement with the observations.

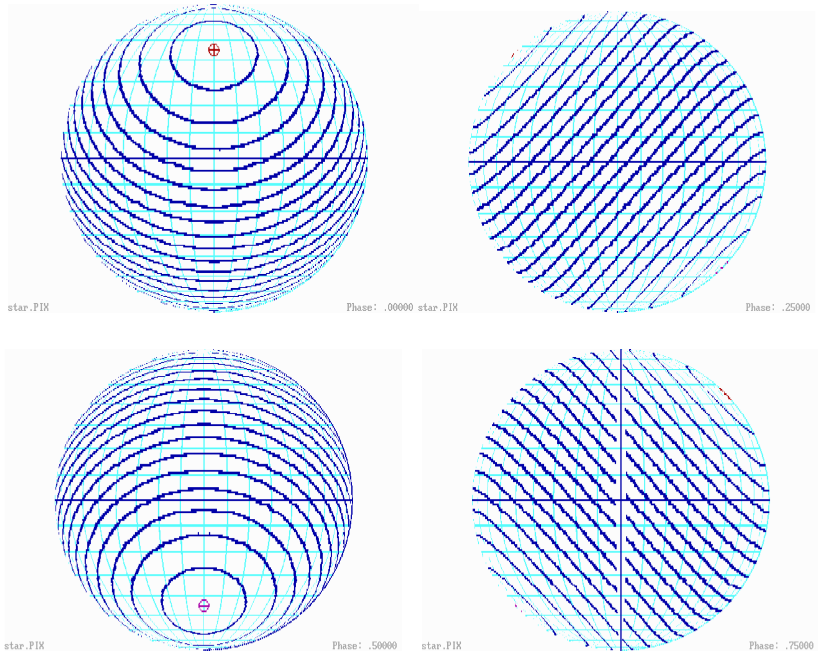
Geometrical conditions:



The star is orientated in the Cartesian coordinate system with its rotation axis coinciding with the z-coordinate. The observer looks at the axis by the inclination angle *i*.

Coordination:





Integration:

The visibility of the star by the observer is defined by a window function $w(i, \varepsilon, \delta, \varphi)$ containing

i inclination δ latitude

ε limb darkening φ longitude ,

which averages and normalizes the vector $\boldsymbol{B}(r, \delta, \varphi)$ with the orthogonal components B_r, B_φ, B_δ of the magnetic surface field:

$$\boldsymbol{B}_{\text{int}}(t) = \frac{\int\limits_{\delta=-\pi/2}^{\pi/2} \int\limits_{\varphi=0}^{2\pi} \boldsymbol{B}(r,\delta,\varphi) w(i,\varepsilon,\delta,\varphi - t) \mathrm{d}\varphi \mathrm{d}\delta}{\int\limits_{\delta=-\pi/2}^{\pi/2} \int\limits_{\varphi=0}^{2\pi} w(i,\varepsilon,\delta,\varphi - t) \mathrm{d}\varphi \mathrm{d}\delta}$$

This integral formula gives the integrated mean of the disk seen by the observer and comprises the *convolution integral*, representing the rotation with its map $\boldsymbol{B}(\varphi, \delta)$ behind the window $w(i, \varepsilon, \delta, \varphi)$.

The line profile is found by a further integration. The distribution of the polarized radiation over a region b around $\boldsymbol{B}$ is defined by $\omega(b)$ and convoluted with the phase integral equation:

$$\boldsymbol{B}_{\text{int}}(t,b) = \frac{\int\limits_{\delta=-\pi/2}^{\pi/2} \int\limits_{\varphi=0}^{2\pi} \int\limits_{-\infty}^{+\infty} \boldsymbol{B}(r,\delta,\varphi,\beta) w(i,\varepsilon,\delta,\varphi - t) \omega(\beta - b) \mathrm{d}\beta \mathrm{d}\varphi \mathrm{d}\delta}{\int\limits_{\delta=-\pi/2}^{\pi/2} \int\limits_{\varphi=0}^{2\pi} \int\limits_{\infty}^{+\infty} w(i,\varepsilon,\delta,\varphi - t) \omega(\beta - b) \mathrm{d}\beta \mathrm{d}\varphi \mathrm{d}\delta}$$

This equation is the analytical representation of the geometric line profile at a given phase, containing the convolutions due to rotation and to the frequency distribution of the field strength.

Accounting for the distribution of elements on the surface the magnetic map $\boldsymbol{B}(r, \delta, \varphi)$ will be replaced by the product $f(r, \delta, \varphi)$ $\boldsymbol{B}(r, \delta, \varphi)$ with $f(r, \delta, \varphi)$ as the coordinated factor. (www.ewald-gerth.de/112.pdf)

Spectral lines:

All **information** about the **magnetic field** of a star
is contained in the Zeeman-displaced line profiles
originating from its atmosphere.

For full comprehension there has to be taken into account:

1. the atomic processes of absorption and emission,
2. the radiation transfer through the stellar atmosphere
3. the geometric conditions –
   a. the projection at the sphere to the line of sight,
   b. the stratification of the atmospheric layers,
   c. the topographic arrangement
   of the observable field, –
4. the integration over the star’s visible disk,
5. the reconstruction of the intrinsic magnetic field.

Information loss:

The really **observed integral radiation** emerging from the star is a result of vast processes of mixing, averaging, and convolving with an overwhelming amount of parameters.

The principal lack of information because of partial **invisibility** of the star's sphere impedes the analysis of the origin of the magnetic field (the "ill-posed" inversion problem – Khokhlova, 1986).

The analysis of the element distribution was tackled by "**Magnetic Doppler Imaging**" (Piskunov), using the Doppler-shift of element-determined lines caused by the star's rotation in presence of a magnetic field.

Because of the **complexity of the stellar magnetic field**, the pioneers of stellar magnetism stuck only to the observable *effect* and called the longitudinal component the ***effective magnetic field*** $\boldsymbol{B_{\mathrm{eff}}}$.

Surface distribution:

A havy complication for analysis is the distribution of the **chemical elements** over the star's surface.

Only for **hydrogen** the distribution is assumed to be nearly equal.

The better measurable metallic lines are bound to elements, the distribution of which is uncertain.

However:

The **distribution of elements** is not random at all.

Theoretical considerations suggest, that the atoms are settled by their magnetic and electric properties **around the magnetic poles** in form of **caps and rings** – from outside by **accretion** and from inside by **diffusion**.

Programming:

Modelling stellar magnetic fields outgoing from sources in a **straight-forward calculation** was outlined by Gerth, Glagolevskij, and Scholz (1997, 1998) and realized by computer program on the base of vector algebra.

The magnetic field **vector consists of three components** with the unity vectors in direction of the radius of the star (normal vector), in direction of the longitude (î -vector), and in direction of the latitude (δ-vector). A **fourth component** is added for a scalar magnitude, which can be used for different purposes (brightness, transparency, factor).

Here we use the **fourth component as the factor**, which is related to the topographic element density on the surface, represented as a cartographic map.

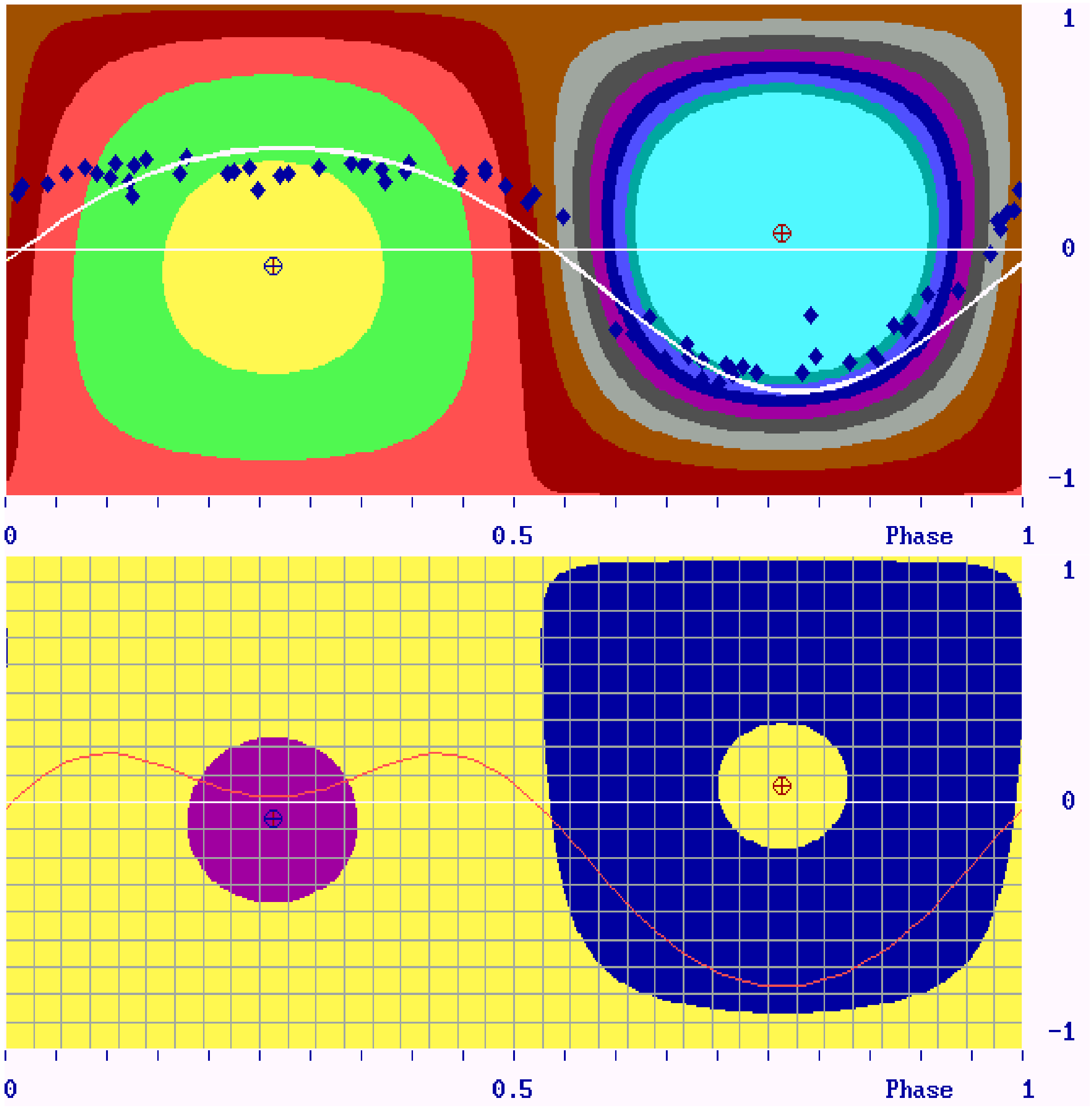
Application:

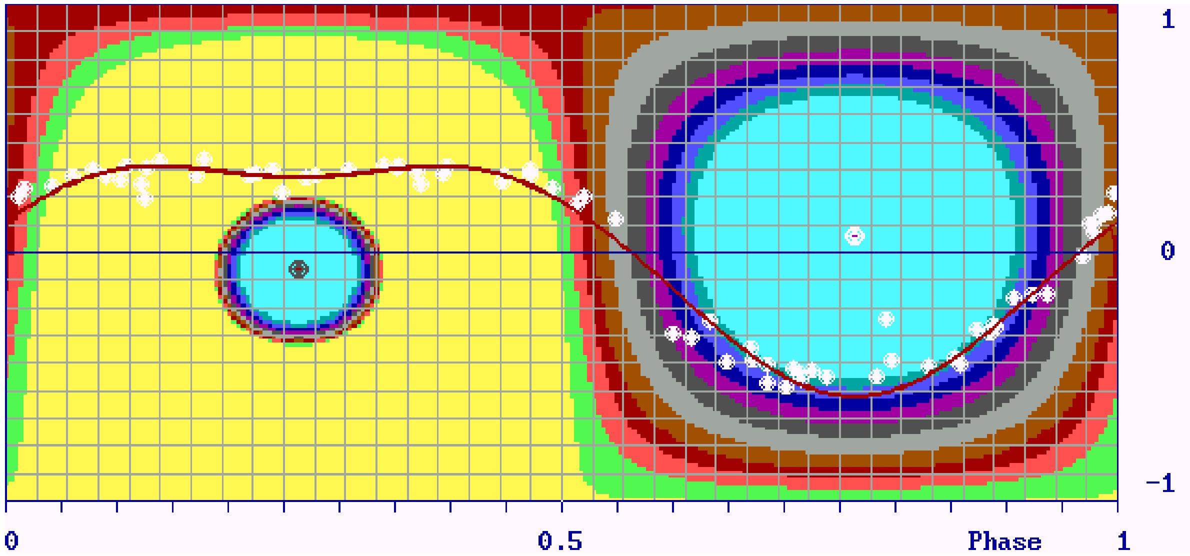
On a poster shown on this conference we demonstrate the effect of the element distribution as phase curves of $\boldsymbol{B}_{\mathrm{eff}}$ – applying especially to the well-investigated star 53 Cam (Gerth, Glagolevskij 1997, 2000).

One can see that Babcock’s famous pioneering measure-ments of metallic lines are agreeable with a model of a decentered magnetic dipole and partly covered poles.

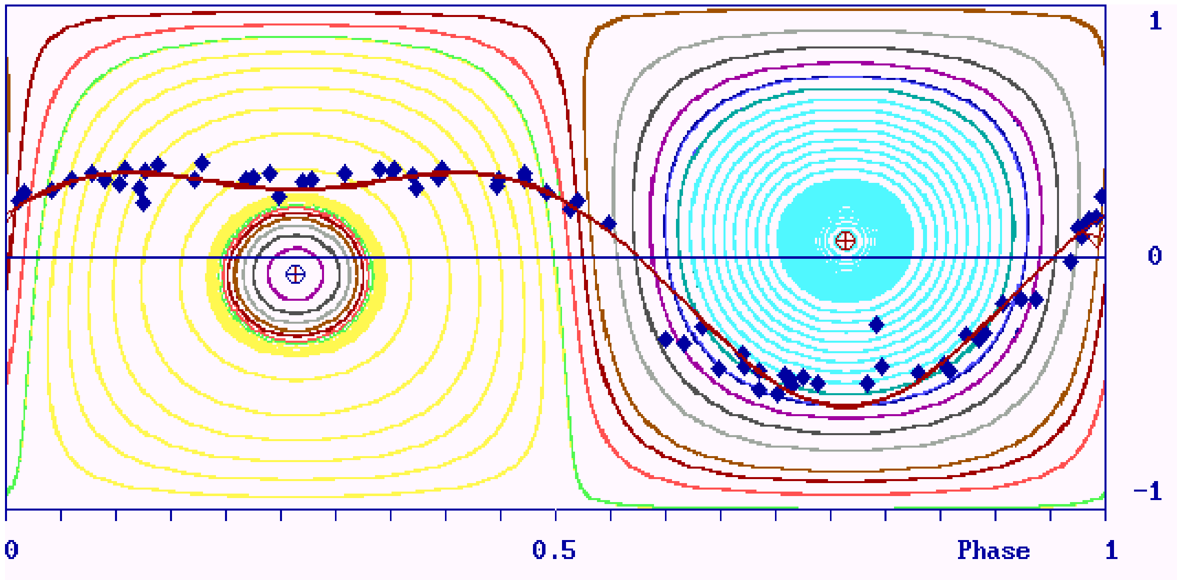
Obviously, the field-relevant metallic elements are **distributed in rings around the poles** –
for this gives the best fit of the observational data to the proposed **sources-rings-model**.

Exempli gratia:





1
0
−1
0
0.5
Phase
1



1
0
−1
0
0.5
Phase
1

Conclusions:

The observed magnetic field is an **integrated** one, called the ***integral magnetic field*** $\boldsymbol{B}_{\mathrm{int}}$, which exists independently of visibility and detection.

The integration is not related to the magnetic field itself but to the information transferring medium: the **spectral line profile**.

The **element distribution** acts like a transparency filter for the field. Accounting for all integration, projection, polarization, and measuring effects, we measure the ***effective magnetic field*** $\boldsymbol{B}_{\mathrm{eff}}$.

References

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Papers and posters of the authors are available from the homepage: **`www.ewald-gerth.de`**