

Fig. 1. a) Magnetic Mercator map of a central dipole with an obliqueness angle  $\beta = 45^\circ$ . The colors in a progressive arrangement mark iso-magnetic areas around the magnetic poles. b) Group of phase curves with the parameter  $i$  (inclination angle). Step:  $i = 10^\circ$ . All phase curves show sinusoidal forms, which can be taken as the characteristic feature of a central dipole. The phase curves correspond to the topographic arrangement of the map, which is suggested by the underlaid colored coordinate net. Red: Phase curve looking equator-on ( $90^\circ$ ). c) Symbolic arrangement of the dipole with a distance of the point-like sources of  $l = 0.01 R$ . The meaning of the details is valid also for the following figures, with only the special things.

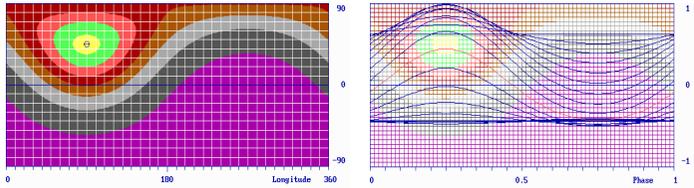


Fig. 2. a) Magnetic Mercator map of a decentered dipole by  $0.25 R$  in direction of the magnetic axis. The iso-magnetic areas show a significant deviation from symmetry. The polar strength differ from each other and can be calculated only for a dipole axis pointing to the center by the formulae  $B_{p1} = C (r/R)^3 \sin^2 i$  and  $B_{p2} = C (r/R)^3 \cos^2 i$  – holding also for external dipoles. b) Group of phase curves analog to Fig. 1b. Step:  $i = 10^\circ$ . Red: inclination angle  $i = 90^\circ$ . All phase curves show deviations from the sinusoidal form, due to the decenteration of the dipole. Nevertheless, the map and the phase curves correspond in polarity to each other. c) Symbolic arrangement of the dipole like in Fig. 1 but shifted by  $0.8 R$ .

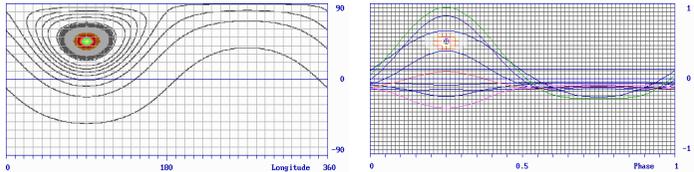


Fig. 3. a) Magnetic Mercator map of a decentered dipole by  $0.75 R$  in direction of the magnetic axis. The iso-magnetic areas show a further deviation from symmetry. We use now iso-magnetic lines in order to show the detailed structure in the iso-magnetic areas. b) Group of phase curves. Step:  $i = 20^\circ$ . Inclination: Red  $i = 90^\circ$ , Green  $i = 40^\circ$ , Violet  $i = 120^\circ$ . The phase curves are deviating heavily from the sinusoidal form. Interesting is the reversal of the polarity of the integral field strength for higher angles  $i$ . This is due to a large visible part of the surface, where the lines of force return and penetrate the surface from outside. c) Symbolic arrangement of the dipole like in Fig. 2 but shifted by  $0.8 R$ .

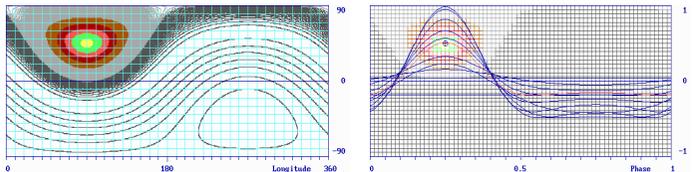


Fig. 4. a) Magnetic Mercator map of a decentered dipole by  $2 R$  in direction of the magnetic axis outside the star. The iso-magnetic lines show a similar picture like the dipole being located inside the star – see Fig. 3. Even the polarity has not changed. In reality, the polarity has changed twice, because 1. the negative pole of the dipole points to the star's surface and 2. The lines of force penetrate the surface from outside. b) Group of phase curves. Step:  $i = 10^\circ$ . Inclination: Red  $i = 90^\circ$ . The phase curves are similar but show no reversal of the polarity of the integral field strength for higher angles  $i$ . c) Symbolic arrangement like in Fig. 2 but shifted by  $2 R$  – outside the star! The external magnetic source is not fixed to the star's body. However, this could be a magnetic body in the neighborhood – maybe an orbiting magnetic companion.

# The decentered magnetic dipole – a challenge for modelling ?

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The *magnetic dipole* reveals itself by modulation of the observed integral magnetic field strength in the rotational course of the star as an *oblique rotator*, defining the magnetic poles as topographic features fixed at the surface of the star. Assuming a central dipole, the phase curve of the integral magnetic field strength would be a sinusoidal one. The obviously existing deviations from the sinusoidal form J. D. Landstreet in 1970 explained by the heuristic assumption of a *decentered magnetic dipole*, which enables one to describe most of the observed magnetic phase curves rather well. In 1954 A. J. Deutsch was the first to represent the surface field analytically by a truncated expansion of spherical harmonics, which can be coordinated to *central magnetic dipoles*, quadrupoles, etc. Also, the description of the surface field of a decentered dipole would require such an expansion. However, since for every degree  $n$  the number of parameters grows with  $2n+1$ , we have for a dipole 3 parameters, for dipole+quadrupole 8 parameters, etc. Truncating after the octupole term yields 21 parameters. For modelling, these *parameters* are taken as *generating magnitudes*, the physical meaning of which is not defined.

**This is, indeed, a challenge for modelling!**

We recommend for the calculation of the magnetic dipole field the straight-forward way from the origin of the *magnetic dipole source* as a *generating magnitude* on the physical base of the potential theory and the mathematical base of vector algebra.

This is the foundation of modelling stellar magnetic field structures by the method of the *Magnetic Charge Distribution (MCD)*, proposed by the authors in former publications :

Gerth, E., Glagolevskij, Yu.V. 2000, in *Stellar Magnetic Fields* (eds. Yu.V. Glagolevskij and I.I. Romanyuk), Nizhnyj Arkhizy, 151  
 Gerth E., Glagolevskij, Yu.V. 2001, in *Magnetic fields across the Hertzsprung-Russell diagram* (eds. G. Mathys, S.K. Solanki, D.T. Wickramasinghe), Santiago de Chile, 248, 333

References to A.J. Deutsch and J.D. Landstreet:  
 Deutsch A.J., 1954, *Astroph. J. TIAU*, Cambridge University Press, 8, 801  
 Landstreet J.D., 1970, *Astroph. J.*, 159, 1001

## What is a *decentered* magnetic dipole in a star?

- The decentered magnetic dipole reveals itself by two *asymmetric magnetic hemispheres* of opposite polarity at the star's surface.
- Despite the physical fact, that the magnetic field with closed lines of force is generated by an electric current circulating in the star's interior, the field lines penetrating the surface can be traced back approximately to point-like *virtual sources* bearing "magnetic charges".
- Two magnetic charges,  $Q_1 = +Q$  and  $Q_2 = -Q$ , of equal quantity but opposite polarity and a spatial distance  $l$  constitute the axial vector of the magnetic moment  $M = Ql$ , which is the *generating magnitude* of the characteristic spatial dipole field.

## Why do *asymmetric* stellar magnetic surface fields occur?

- The star might have a history with *disturbances* (impacts, outbursts, solar-like star-spots, etc.).
- The star is affected by a cosmic neighbor in a binary system, which distorts the structure of its own magnetic field, or/and it is externally influenced by the *interaction* with a self-magnetic companion (e.g., white dwarf, black hole, cataclysmic system, etc.).
- The observed magnetic field strength is measured by the Zeeman-displacement of the spectral lines of the light-absorbing elements in the atmosphere, which are normally distributed *inhomogeneously* over the star's surface, covering also the magnetic poles differently.
- The surroundings of the star might be partly obscured by clouds or an eclipsing big companion.

## How to *analyze* the observed magnetic surface distribution?

- The direct *inversion* gives an analytical representation - without interpretation of the origin.
- *Modelling* with hypothetical parameters and fitting to the observed facts by a straightforward calculation on grounds of a simplifying but reasonable theory is possible in any case.
- A computer program for "magnetic modelling" should relate to a minimum of parameters to have varied for fitting, which is guaranteed by using *generating magnitudes* (the sources of the field) but not derived or ambiguous ones (e.g., the field strengths at the poles).
- The *parameters* can be regarded as *characteristic magnitudes* of a star, which describe at once (by the corresponding algorithm) the whole magnetic field and can be refined by further observation.

## Is the decentered dipole *restricted* to asymmetric star fields?

- Not at all! The decentered magnetic dipole is a generating magnitude itself and can be combined to a conglomerate of numerous *elementary dipoles* by superposing all single fields to a general one (so as a metallic iron magnet consists of a vast amount of molecular magnets with a common field).
- Each *elementary dipole* is *decentered* relating to the zero-point of the coordinate system.
- The magnetic field of the decentered dipole(s) *fills the entire space* and can be calculated for any plane - so as the surface of a sphere, whereas spherical harmonics are restricted to the sphere.
- Any magnetic field structure can be represented by a *sum of the fields of elementary dipoles*, giving thus a counterpart to an expansion of spherical harmonics, but being much more *general* and physically founded, because it relates to the origin of the magnetic field – its *sources*.

**The decentered magnetic dipole should *not* be a challenge!**

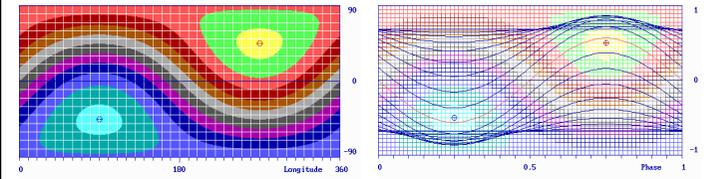


Fig. 5. a) Magnetic Mercator map of a central dipole with an obliqueness angle  $\beta = 45^\circ$ . In comparison with Fig.1, the map is shifted along the longitude by  $180^\circ$ . This is because of the direction of the dipole perpendicular on the direction of the magnetic moment in Fig.1. For a central dipole the polar field strengths is calculated easily by the formula  $B_p = C R^3$ . b) Group of phase curves with the parameter  $i$ . Step:  $i = 10^\circ$ . Red: Phase curve looking equator-on ( $i = 90^\circ$ ). c) Symbolic arrangement of the dipole with a distance of the point-like sources of  $l = 0.01 R$ .

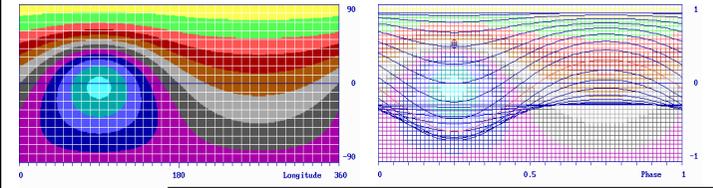


Fig. 6. a) Magnetic Mercator map of a perpendicularly decentered dipole shifted to  $r = 0.2 R$ . In comparison with Fig.2, the map is totally inverted and asymmetrical. The computation of the polar field strengths cannot be done by simple formulae like those given in the legend of Fig.2. This problem is easily overcome by application of the computer program used for the graphics. b) Group of phase curves with the parameter  $i$ . Step:  $i = 10^\circ$ . Red: Phase curve ( $i = 90^\circ$ ). The phase curves show the asymmetric forms, which indicate a decentered dipole (see Fig. 2). c) Symbolic arrangement of the dipole with a distance of the point-like sources of  $l = 0.01 R$  perpendicular to the direction of the radius.

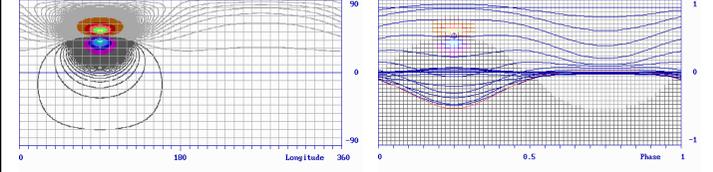


Fig. 7. a) Magnetic Mercator map of a perpendicularly decentered dipole shifted to  $r = 0.8 R$ . In comparison with Fig.3, the map is also inverted and asymmetrical. The flat dipole under the star's surface, however, produces a pair of close poles, which resembles to a sunspot. This offers the possibility of modelling the global structure of solar-like starspots. b) Group of phase curves with the parameter  $i$ . Step:  $i = 10^\circ$ . Red: Phase curve ( $i = 90^\circ$ ). The phase curves show the typical asymmetric forms of a decentered dipole (see Fig. 3). c) Symbolic arrangement of the dipole with a distance of the point-like sources of  $l = 0.01 R$  perpendicular to the direction of the radius by  $0.8 R$ .

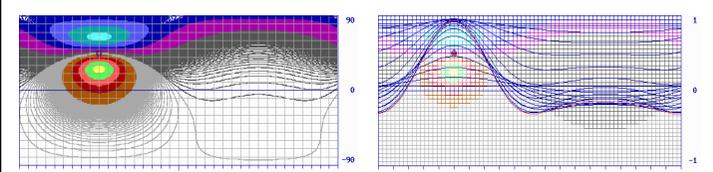


Fig. 8. a) Magnetic Mercator map of a decentered dipole by  $2 R$  perpendicular to the direction of the radius outside the star. The iso-magnetic lines show also the spot-like but another picture than Fig. 7, because the polarity has changed crossing from inside to outside the star. The lines of force penetrate the surface in the opposite direction. b) Group of phase curves. Step:  $i = 10^\circ$ . Inclination: Red  $i = 90^\circ$ . The phase curves show a dip in the negative region of the phase curve, which occurs in several magnetic stars. We do not exclude externally influenced magnetic stars. c) Symbolic arrangement like in Fig. 6 but shifted by  $2 R$  – outside the star! The external magnetic source could belong to an orbiting companion. For a correct computation, however, the orbital motion with a constant angular momentum has to be taken into account. How could this be done using the star-bound spherical harmonics ?!