On generating and derived magnitudes of stellar magnetic fields

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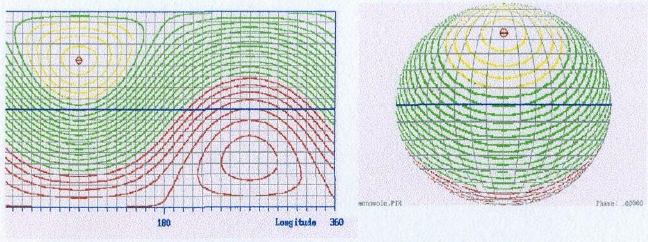
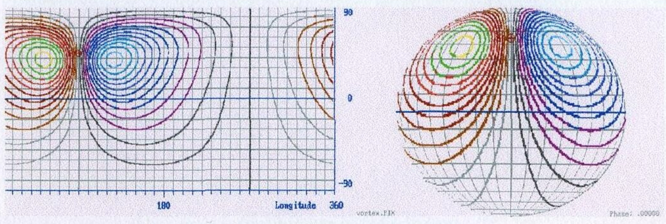
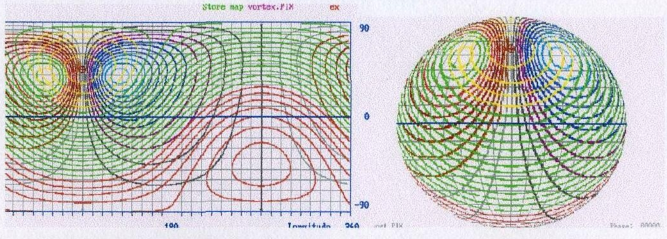


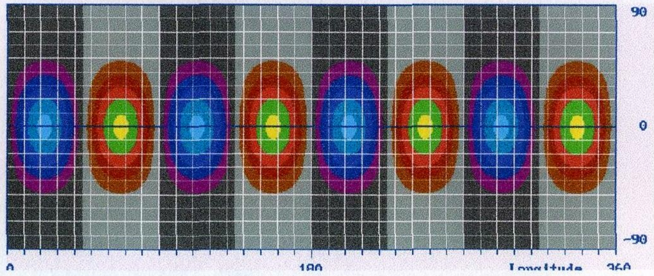
Magnetic monopole. $\varphi = 90°$ $\delta = 45°$ $r = 0.5$ Mercator map and globe



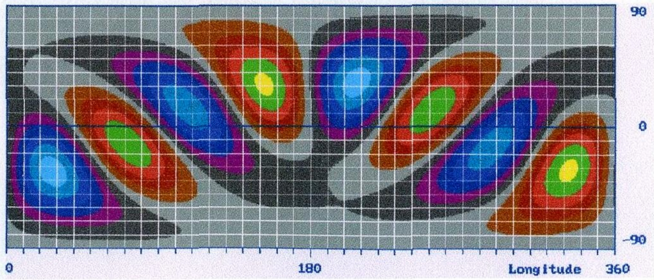
Magnetic vortex. $\omega = 90°$ $\delta = 45°$ $r = 0.5$ $\varepsilon = 1$ $\vartheta = 0$



Combination of the maps of a source and the vortex (no field superposition!)



Calculation of the magnetic surface field by means of spherical harmonics. Octupoles in the equatorial plane (top) and tilted by 40° (bottom)



Modeling of magnetic stars goes out from the **generating** magnitudes and is a matter of **construction** by a strategy of **forward calculation.**

The model is **fitted** to the appearance of the real object by **variation of parameters** and optimizing.

Parameters cannot be taken from the **appearance** of the object. Magnetic **poles** with their field strengths and coordinates can **not** serve as **parameters.**

The magnetic field strength on the star's surface is a **derived magnitude.**

The magnetic field is a vector field, which is determined by its **sources** and **vortices,** which are the **generating magnitudes** as parameters.

The **Magnetic Charge Distribution** (MCD) method uses arrangements of magnetically charged sources for the construction of **static magnetic fields.**

Combinations of sources - so as the **magnetic moment** of a **magnetic dipole** - are also generating magnitudes.

The generating magnitudes **sources and vortices** represent the **eigenvalues** as solutions of the differential equation of decentered spherical potentials.

Solution by spherical harmonics $P_p^{\,q}$ of *Legendre*'s differential equation:

$$(1-z^2)\frac{d^2u}{dz^2} - 2z\frac{du}{dz} + \left(p(p+1) - \frac{q^2}{1-z^2}\right)u = 0$$

"associated *Legendre* functions of first kind"
p degree of *Legendre* polynomial
$q = -p \cdots +p$ order index

Comparison with the oscillation differential equation:

$$M\frac{d^2x}{dt^2} - 2R\frac{dx}{dt} + Dx = 0$$

Solution by eigenfrequencies: $\omega = \sqrt{\dfrac{D}{M} - \dfrac{R^2}{M^2}}$

Generating magnitude: *D/M*. (*D* direction force, *M* mass, *R* resistance)

Common properties of spherical harmonics and trigonometric functions:

1. Formulation as differential equations
2. Reduction to eigenvalues
3. Solution as expansion of functional terms (functions of eigenvalues – eigenfunctions)
4. Linearity and orthogonality, superposition
5. Transformation to the complexe projection space (*Laplace*-Transformation)
6. Inverse analysis procedures
   a) *Fourier* analysis
   b) "*Legendre*" analysis

The MCD-method uses the intrinsic eigenvalues as the original **generating magnitudes** of the magnetic field.

For the **computation** of the magnetic field, standard algorithms for the linear **differential operators** grad and curl are developed.

1. Monopole:

$$\text{grad}\, U = \frac{\partial U}{\partial x}\mathbf{i} + \frac{\partial U}{\partial y}\mathbf{j} + \frac{\partial U}{\partial z}\mathbf{k}$$

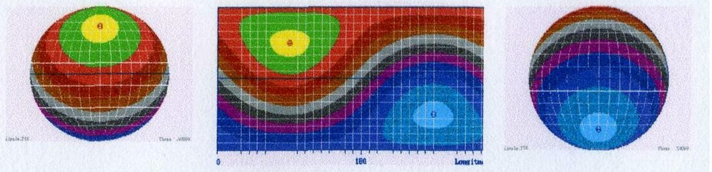
2. Vortex:

$$\text{curl}\, I = \left(\frac{\partial I_z}{\partial y} - \frac{\partial I_y}{\partial z}\right)\mathbf{i} + \left(\frac{\partial I_x}{\partial z} - \frac{\partial I_z}{\partial x}\right)\mathbf{j} + \left(\frac{\partial I_y}{\partial x} - \frac{\partial I_x}{\partial y}\right)\mathbf{k}$$

x, y, z coordinates; $\mathbf{i}, \mathbf{j}, \mathbf{k}$ Cartesian unit vectors; U potential, I electrical current components I_x, I_y, I_z

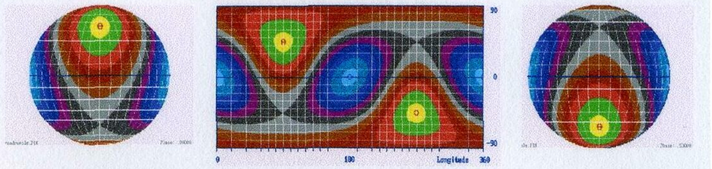
The **linearity** of the differential operators enables the superposition of a multiplicity of singular source fields in a successive arithmetic procedure.

The main generating magnitude is the **magnetic dipole moment,** which is the **elementary** brick to build any magnetic body with its surrounding field.

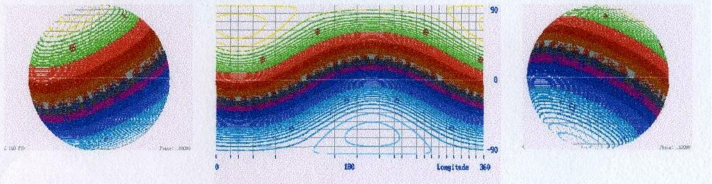


Central magnetic **dipole** with separated magnetic charges

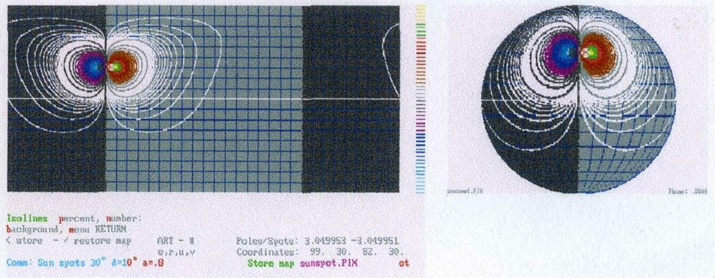
Parameters:	Radius-fraction	Longitude	Latitude	Charge
	$r_1 = 0.5$	$\varphi_1 = 90°$	$\delta_1 = 45°$	$Q_1 = +1$
	$r_2 = 0.5$	$\varphi_2 = 270°$	$\delta_2 = -45°$	$Q_2 = -1$



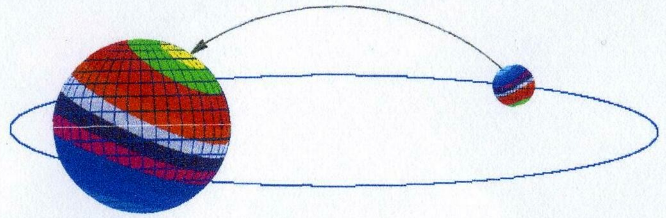
Magnetic **quadrupole** of two decentered dipoles with antiparallel magnetic moments tilted to the equatorial plane by 45°



Magnetic **multipole** as superposition of the fields of 80 dipoles, arranged in a circular magnetic sheet with tilting of 30° to the equatorial plane.



Extremely decentered dipole. Simulation of the global field if a solar-like spot



External dipole. Scheme of the opposition of a companion in a binary system, which influences the main star with its field (suspected at the CP star ν Cep).

Poster representation at the Conference on *MAGNETIC STARS* in Nizhnij Arkhyz, Russia, 27-31 August 2003