

## A quadrupole model of the magnetic field of $\beta$ CrB

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**Abstract.**<sup>1)</sup> A model of the magnetic field of the star  $\beta$ CrB is constructed by the method of the “Magnetic Charge Distribution”. The magnetic field is described by superposition of two decentered dipoles, which are arranged in the equatorial plane perpendicularly to the axis and oppositely directed to the center of the star. The dipoles form an *irregular quadrupole* and produce on the surface four magnetic spots with a maximal magnetic field strength at the poles of  $B_p = 14.5$  kG. The angle of declination of the rotation axis to the line of sight has been ascertained by the quadrupole model to  $i = 13^\circ$ . In consideration of all aspects the idea is advanced of very strong influence of an inhomogeneous distribution of chemical elements onto the measurable phase curves of the effective magnetic field. Apparently, the star has passed a convective phase in an early stage of evolution.

**Key words:** Chemically peculiar stars, magnetic fields,  $\beta$ CrB

### 1. Introduction

The chemically peculiar CP-star  $\beta$ CrB (HD 137909, F0p) is one of the most investigated magnetic stars because of its brightness and sharpness of the spectral lines. STIBBS (1950) and DEUTSCH (1954, 1958) explained the variability of the observed integral magnetic field strength with a period of 18.497 d (STEINITZ 1964, 1967) by a model of an obliquely rotating magnetic dipole, which is rigidly bound to the stellar body and might be shifted off the center of the star (LANDSTREET 1970). DEUTSCH (1970) was the first to describe the typical spherical asymmetry by moments of dipoles, quadrupoles etc. of spherical harmonics.

An early attempt to explain the phase relation of the effective field  $B_e$  by an *equatorially symmetric rotator model* has been done by OETKEN (1977). She derived from the shape of the phase curve a prevailing quadrupole moment over the dipole moment by  $u_{20}/u_{10}=2.0$ .

Modelling of the magnetic field of  $\beta$ CrB has been done also by BAGNULO et al. (1998, 2000, 2001), who describe the stellar surface field by a *second-order expansion* of spherical harmonics on the base of a centered dipole plus a non-linear quadrupole.

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In the present article the magnetic field structure of  $\beta$ CrB is analyzed by the modelling method of the “*Magnetic Charge Distribution*” (MCD), which is described by GERTH & GLAGOLEVSKIJ (2000) and applied to real stellar objects by GERTH et al. (1998, 1999, 2000, 2001, 2002).

The *magnetic charges* are treated analogously to the *electric charges*. This is advantageous for the computation of the spherical field of a point-like source. The idea of the construction of the stellar magnetic field out of its generating sources has been adopted by KHALACK et al. (2001, 2002, 2003), who came to the same conclusions as we did before.

## 2. Modelling by the MCD-method

The modelling method of the “*Magnetic Charges*” gives a description of the magnetic vector field on the star’s surface outside and inside the star. So it would be possible to trace the lines of force back from the surface somewhat into the interior, keeping in mind, however, the limitations of the model in the vicinity of the sources. Sources outside the star, like orbiting magnetic bodies of a binary system, can influence their magnetic field onto the atmosphere on the surface of the main star.

The MCD-method relates to single field sources with “magnetic charges”, which are surrounded by spherical magnetic fields. Such sources might be distributed anywhere in the space. A spherical body like a star shows to the observer its magneto-informative surface, which is penetrated by the lines of force. Since single *magnetic charges* do not exist in reality, we take them as *virtual sources* - in analogy to the virtual light source in optics, from which the light seems to diverge (after a mirror or a lense). A couple of two oppositely charged sources, however, is physically relevant as a *magnetic dipole*. The fields of numerous dipoles superpose linearly to irregularly arranged *quadrupoles* and *multipoles* rendering all possible field structures. Such multipoles are arrangements of field sources and should not be mistaken as a derivation of the *Legendre* coefficients in a series of spherical harmonic functions.

The vectorial field of a point-like source is calculated for any point in the surrounding space and, therefore, for any geometrical body like the sphere of a star, including its anywhere positioned surface - in contrast to the calculation of the field by means of spherical harmonics (see OETKEN 1977, BAGNULO 2000, 2001), which relates only to the surface of a sphere. This means, that the calculation of a decentered or an outside the star located dipole cannot be performed by spherical harmonics.

A special standard algorithm calculates the field strength as the gradient of the potential of the charged source - making dispensable the use of *Legendre's* spherical functions for MCD-modelling of stellar magnetic fields.

## 3. The observational results used for modelling

The phase relations of the effective (mean longitudinal) magnetic field strength  $B_e$  and the surface (mean field modulus) magnetic field strength  $B_s$  taken by

some authors differ heavily. This is shown in the paper of Leone & CATANZARO (2001), where different phase curves are drawn together. The curves show systematic deviations up to 1 kG, although the inner accuracy of the measurements by any of the authors is essentially higher. Thus, one has to be cautious to compile simply the data. We relate the modelling only to homogeneous data sets.

From all available observational data we chose the phase ( $P$ ) relations  $B_s(P)$  from WOLFF & WOLFF (1970) and MATHYS et al. (1997) (Fig 1b, 2a). The first data are taken from old photographic observations by the lines of Cr, Ti, and Fe. The second data were obtained by a line of Fe II (6149.2 Å) with a CCD-camera. Further we employ the phase curves  $B_e(P)$  with magnetic field measurements of the hydrogen line  $H_\beta$  by BORRA & LANDSTREET (1980) (Fig. 1a, 2b). A third data set was taken by WADE et al. (2000) using Fe-lines, which will be analyzed separately and compared with the other results (Fig. 2c).

All observational phase curves are arranged using the ephemeris after KURTZ (1989)

$$\text{JD}_{\text{magnetic max}} = 2434204.70 + 18.4868 \text{ E.}$$

Relating to the representation of the observational data in Fig. 1 and Fig. 2, let us consider some properties of the phase relations that have an effect on the model of the magnetic field in the case of  $\beta$ CrB. The small amplitude of  $B_s(P)$  leads to a small angle of declination  $i$ . At the other hand, the angle  $i$  effects on the amplitude of the calculated relation  $B_e(P)$ . The inclination angle  $i = 13^\circ$  found by ours corresponds best of all to both phase relations. The small magnitude of  $B_e < 1$  kG together with the large quantity  $B_s \approx 5.5$  kG argues for the assumption, that the magnetic poles are located at the limb of the visible disk, e.g., close to the equatorial plane. The star is visible almost from its pole.

### 3.1. The dipole model

In order to construct a magnetic dipole, the position of the two “magnetic charges” of opposite polarity inside the star is defined by a procedure of successive approximations.

At first we select arbitrarily a magnetic moment  $M = Qr$  ( $Q$  - magnetic charge,  $r$  - its distance from the center of star as fraction of the radius), the longitude  $\lambda$ , the latitude  $\delta$ , and the inclination  $i$  to the line of sight. For the centered dipole model with positive (+) and negative (-) charges by equal distances of the sources from the center yields:

$$r_+ = r_-; \quad \lambda_+ - \lambda_- = 180^\circ; \quad \delta_+ = -\delta_-.$$

The fitting of the observed to the calculated phase relations  $B_e(P)$  and  $B_s(P)$  is carried out by the least squares optimization method. It comes out, that the position of the positive charge in  $\beta$ CrB is  $\lambda = 0^\circ$ ,  $\delta = +1^\circ$ , and that of negative charge is  $\lambda = 180^\circ$ ,  $\delta = -1^\circ$ . The maximal field strength at the poles is calculated for the dipole model to  $B_p = 9.4$  kG. The phase curves derived from the model are shown in Fig. 1 by solid lines. In principle, the dipole model requires two maxima of  $B_s$ , which should pass through a central meridian of the positive and negative magnetic poles. But we see, that maximum and minimum

of the relation  $B_s(P)$  appear in the moment when  $B_e \approx 0$ , instead of the moment when  $B_e$  has its maximum. This consideration and the test calculation show, that the structure of the magnetic field of  $\beta$ CrB cannot be a dipole one – as it was asserted already by BAGNULO (2000), who demanded for the representation of the surface magnetic field of  $\beta$ CrB the addition of a non-linear quadrupole.

### 3.2. The quadrupole model

The disagreement of the computed to the observed phase curves underlying the dipole model shows that  $\beta$ CrB has a magnetic field of complicate structure.

The model, comprising the  $B_s$  and  $B_e$  data from Fig. 1a and Fig. 1b, is calculated using the following parameters:

Table 1

No	$q$	$r$	$\lambda$	$\delta$
1	–	0.3	351	–6.5
2	+	0.3	12	0
3	–	0.3	176	5.3
4	+	0.3	196	0

All charges have the same relative value  $q = 1$ , which is fitted to the observed magnetic field strength on the surface. The magnitude of the magnetic field equals 14.5 kG on the magnetic poles. The distance of the charges from the center is  $r = 0.3$  of the stellar radius. We see in Fig. 3 that both dipoles are arranged more or less in the equatorial plane symmetrically to the center of the star. The inclination angle has been determined by variation of parameters and iterative approximation to  $i = 13^\circ$ .

For comparison we add in Fig. 2 still recent observational data from Wade et al. (2000), which are presented in Fig. 2c. The fitting of the curve could be achieved by a minor variation of the parameter  $\delta$  ( $\delta_1 = -6.6$ ,  $\delta_3 = 5.0$ ), the effect of which proves to be very sensitive.

The distribution of the magnetic field strength over the surface is represented in Fig. 4. The double dipole model describes well the common configuration of the magnetic field, but in detail there may be some differences. So we see in Fig. 2b a small peak in the negative extremum of the observed phase curve  $B_e(P)$ , which is smoothed in the calculated curve. In the paper of Wade et al. (2000) a relation  $B_e(P)$  without such a peak is given, obviously because of the more accurate measurements carried out by new observation and reduction techniques (Fig. 2c). The smoothed phase curves in Fig. 2b and Fig. 2c deviate a bit by the amplitude, the reason of which is not known yet (maybe an instrumental effect). In Fig. 2a, representing the function  $B_s(P)$ , the observations show a broad minimum, which is not contained in the calculated relation. However, the measurements of WOLFF & WOLFF (1970), marked in Fig. 2a by asterisks, contradict to this assertion. Those differences have to be attributed to the uncertainties of the different sets of measurements.

Some significant differences of the curves drawn after the model and the ones obtained by observation can be explained by two reasons:

1) The relation  $B_s(P)$  has been derived from the metallic lines, which are dis-

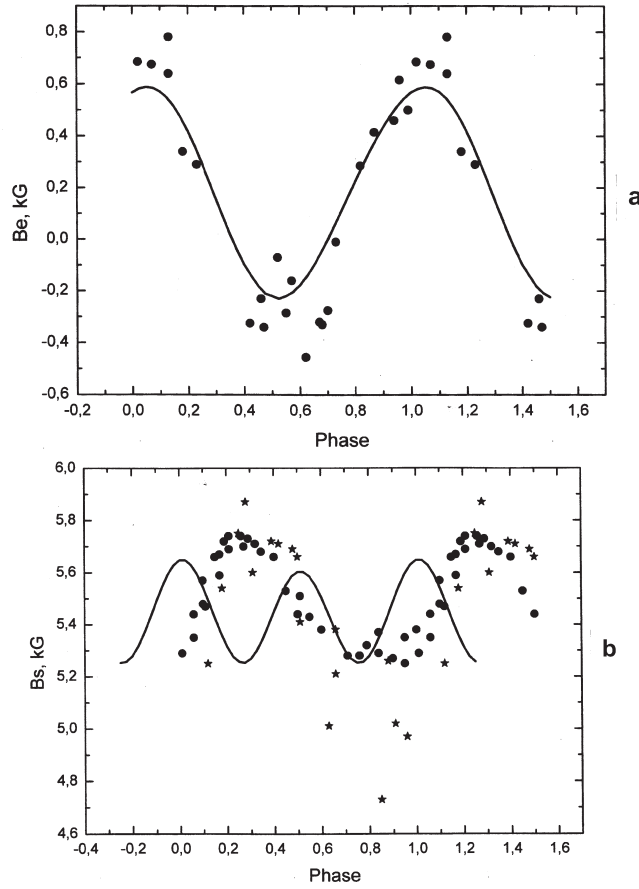


Figure 1. Phase relations for  $\beta$ CrB, applying the dipole model.  
 dots - observations  
 a) effective magnetic field  $B_e$  (Borra & Landstreet, 1980)  
 b) averaged magnetic field on the surface  $B_s$   
 asterisks - data from WOLFF & WOLFF (1970)  
 dots - data from MATHYS (1997)  
 solid lines - calculated phase curves

The modeling of some other stars – for example HD 126515 (GLAGOLEVSKIJ & GERTH 2000) – has proven, that such a situation is possible in the case of an arrangement of two decentered dipoles. But for  $\beta$ CrB a much more complex situation has turned out.

The model of a decentered dipole results in an amplitude of  $B_s$  larger than the observed one. By the method of iterative approximation we found, that both observed curves can be explained assuming the presence of two oppositely directed dipoles, whose magnetic charges are located at the distance of 0.3 stellar radius from the center of the star (see Fig. 3). We call such a combination of two arbitrarily arranged dipoles an *irregular quadrupole*.

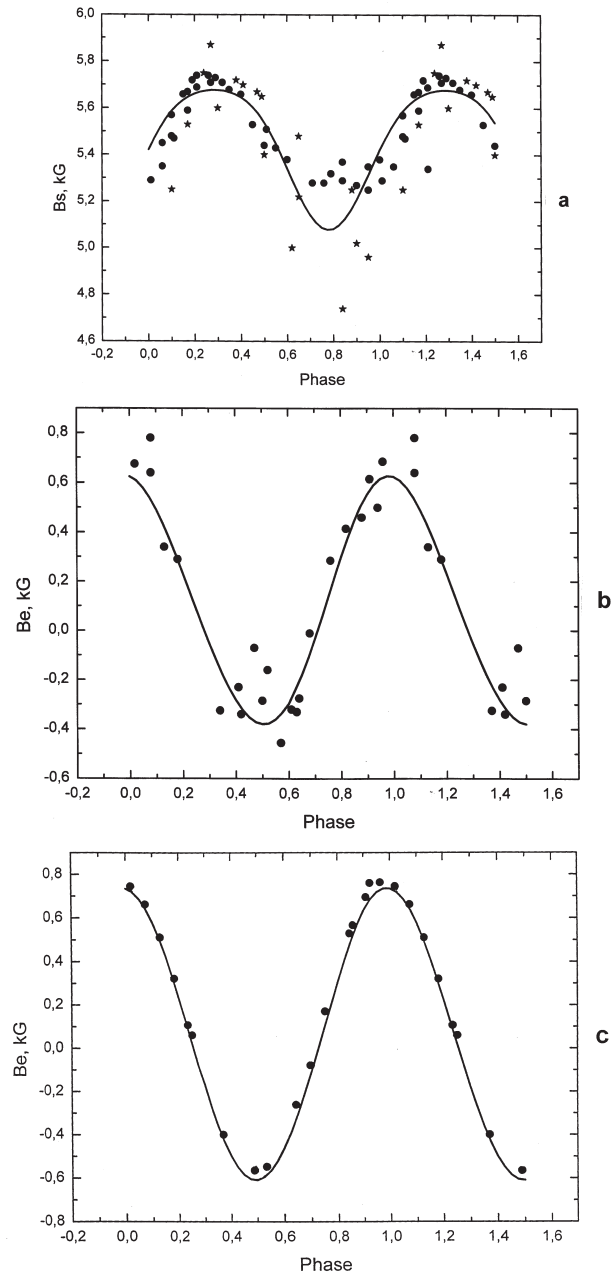


Figure 2. Phase relations for  $\beta$ CrB, using the quadrupole model.

dots - observations

solid lines - calculated phase curves,

a) averaged magnetic field  $\beta$ CrB on the surface  $B_s$

asterisks - data from WOLFF & WOLFF (1970)

dots - data from MATHYS et al. (1997)

b) effective magnetic field  $B_e$  (BORRA & LANDSTREET, 1980)

c) effective magnetic field  $B_e$  (Wade et al., 2000).

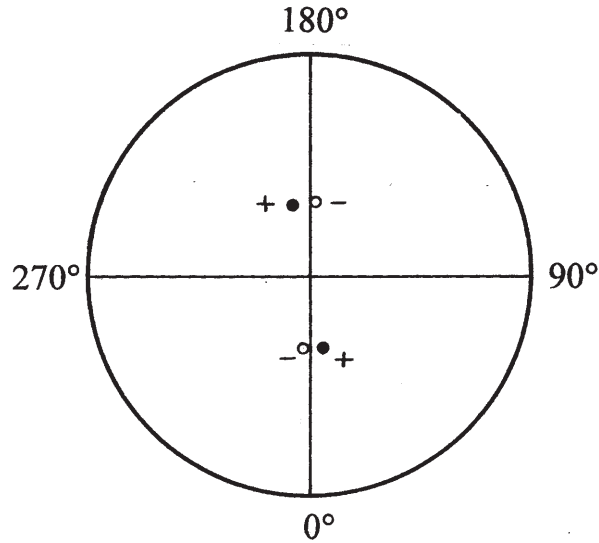


Figure 3. Scheme of the distribution of the magnetic dipoles inside the star (view from the pole)

tributed inhomogeneously.

2) The magnetic field possesses a more detailed fine structure than that one obtained by ours.

The first reason seems to be more probable. The phase curve  $B_s(P)$  in Fig. 2a has a maximum at the phase 0.25 with  $B_e \approx 0$ . Such a situation may be in the only case, when the region between the oppositely arranged poles passes through the meridian and the magnetic charges are located under a small angle  $\lambda$  of longitude.

The configuration found for  $\beta$ CrB consists obviously of two oppositely directed dipoles. Hereby, the axes of the dipoles are not strictly parallel. The large distance between the dipoles of 0.6 of the stellar radius confirms the idea of the existence of two individual dipoles, which combine to a quadrupole. Assuming other distances of the dipoles from the center, the accordance of the calculated and the observed phase relations becomes worse.

The phase relation could be influenced still by other phenomena. Thus, in the papers of WOLFF (1978) and ROMANYUK (1986) some characteristics of the star  $\beta$ CrB were already mentioned, in particular the phase shift of the magnetic curves derived from the lines before and after the BALMER jump.

#### 4. About the gradient of the magnetic field

Our model does not predict a significant gradient of the field as a function of depth. If we assume a thickness of the atmosphere of  $10^3$  km, then the maximal difference of the magnetic surface field on the extreme levels amounts to 25 G.

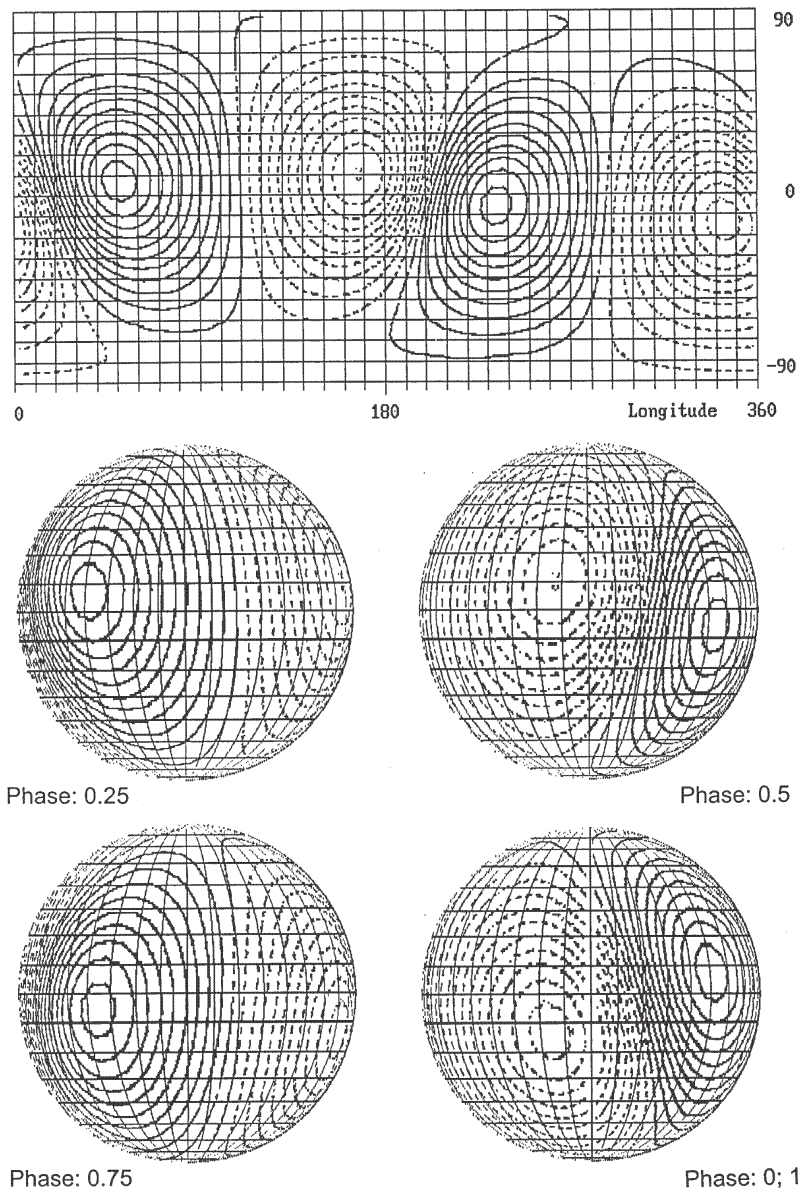


Figure 4. Distribution of the magnetic field strength over the surface represented by iso-magnetic lines of the radial magnetic field strength. Division into 20 equal steps between the positive and the negative poles with the polar strength of  $B_p = 14.5$  kG.  
 Top: Pseudo-MERCATOR map with four magnetic extrema of alternative polarity  
 Bottom: Globes for four rotation phases  
 Solid lines - positive regions; dashed lines - negative regions  
 (Computer program written by E. GERTH)



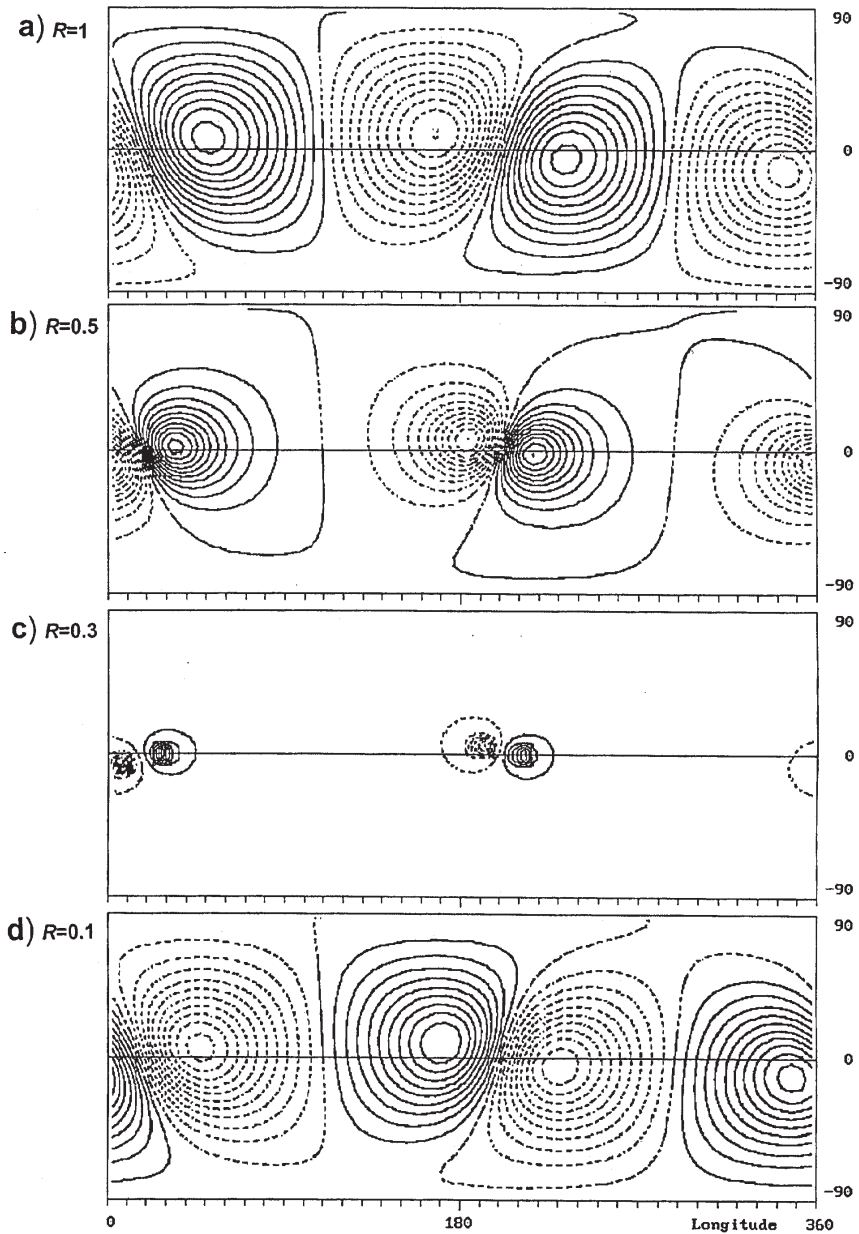


Figure 5. Distribution of the field strength by 4 point-like sources inside the star on different levels of the radius under the surface. The iso-magnetic lines characterize the map on each radial shell between the (internally computed) maximum and minimum of the field strength. Difference divided in 20 steps. Solid lines - positive regions  
Dashed lines - negative regions  
(Computer program written by E. GERTH)

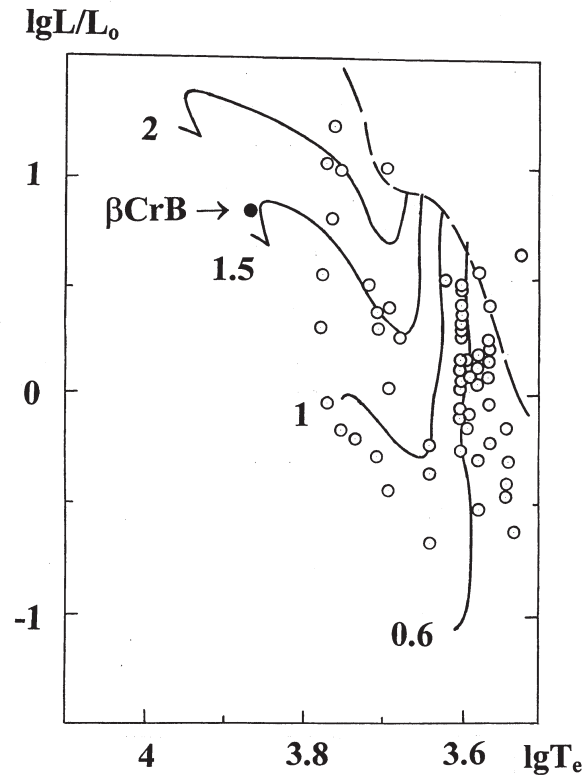


Figure 6. Location of young stars (white circles) of type TTau in the HERZSPRUNG-RUSSELL-diagram and of  $\beta$ CrB (black circle)

This has been calculated with the program used by ours, inserting the radii on the surface and the radius 1000 km higher.

The change of the field structure with depth, starting from the surface, goes at first rather slowly, as we can see in Fig. 5. This result does not contradict to a suggestion given by WOLFF (1978), who assumes a significant gradient of the magnetic field by the depth. The slow course changes strikingly at 0.3 of the stellar radius, where the sources are located at points of singularity. In the infinitesimal vicinity of the singularities the field strength rises up to infinity. Closer to the center the field strength changes the polarity but has a similar structure on an inner sphere at  $0.1 R$ .

This is correct for real point-like sources, as in case of electrical charges. However, the analogy between electrical and magnetic charges has its limitations. The closed magnetic lines of force have no singularity at all. Therefore, the inner structure of the magnetic field is not simply founded on a few magnetic sources. Nevertheless, it demonstrates the continuity of the magnetic field transiting through every shell of the sphere embedded in the surrounding space.

The *magnetic charges as virtual sources* render a powerful heuristic approach for the calculation of the magnetic field structure, which gives an advantageous base for programming and fits the better to the reality the more the distance from the singularities increases.

## 5. Conclusion

We draw the conclusion, that our model – as well as stated in the paper of BAGNULO et al. (2000) – yields a small angle of the rotation axis to the line of sight ( $\approx 13^\circ$ ) and a large angle ( $\approx 90^\circ$ ) to the plane spanned by the dipoles. With the construction of an irregular quadrupole model the contradiction of the calculated  $B_e$ - and  $B_s$ -curves could be overcome.

In present time the hypothesis of a relict origin of the stellar magnetism in CP stars is commonly supported. However, in this case the structure of a field similar to that of  $\beta$ CrB is hardly possible. Such a structure might easier be explained by the hypothesis of a magnetic dynamo. If we locate this star on evolutionary tracks, then we can see that its way passes the early stages of the evolution through regions occupied by T Tau stars; that means, the star could have passed in its early stages a phase of convective instability, when a magnetic field could have been generated. In this case we do not observe at  $\beta$ CrB the original relict field, but a secondary one, generated by the effect of a stellar dynamo. In Fig. 6 the HERTZSPRUNG-RUSSELL-diagram, taken from the paper of PALLA & STAHLER (1994), is demonstrated, in which the evolutionary tracks of early stars, T Tau stars, and that of  $\beta$ CrB are drawn. From this diagram we can see too, that obviously, all CP stars with masses  $M < 2M_\odot$  ( $T_e \approx 9000$  K), i.e. SrCrEu type stars, went in past tense through a convective phase.

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