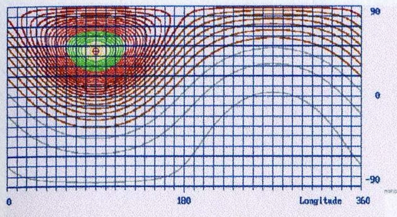
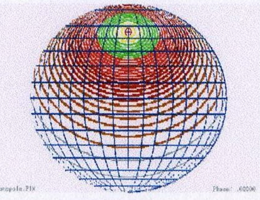
Magnetic Sources





Structure of the field of an eccentric monopole on the surface of a sphere.

The field of a monopole is determined by 4 parameters (3 local coordinates, charge Q):

Radius-fraction	Longitude	Latitude	Charge
$r=0.5$	$\varphi=90°$	$\delta=45°$	$Q=1$

A monopole field source with the charge Q is surrounded by a spherical potential $U=Q/2\pi r$ with r as the radius. The field strength is derived from the potential by the gradient of the potential:

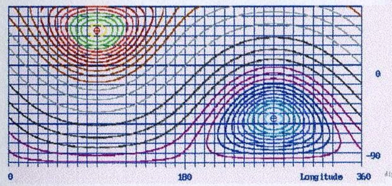
$$\mathrm{grad}\, U = \mathbf{i}\, \partial U/\partial x + \mathbf{j}\, \partial U/\partial x + \mathbf{k}\, \partial U/\partial z$$

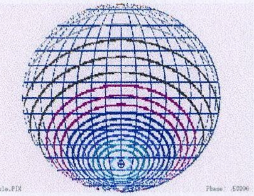
($\mathbf{i}, \mathbf{j}, \mathbf{k}$ – Cartesian unity vectors. Transform to spherical coordinates: $x = r\cos\delta\cos\varphi$, $y = r\cos\delta\sin\varphi$, $z = r\sin\delta$)

For the numerical calculation the spherical potential field a special algorithm has been developed [2], which serves as the fundamental standard for the superposition of numerous monopoles.

Only the electrical monopole field is physically relevant.

However, the algorithm can be used for the magnetic field if oppositely charged monopoles combine to dipoles, quadrupoles, and multipoles [3].





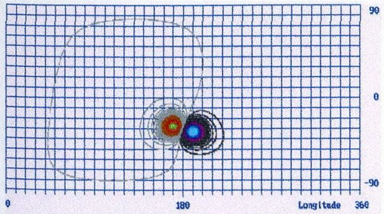
Map and globe of a central magnetic dipole with separated magnetic charges

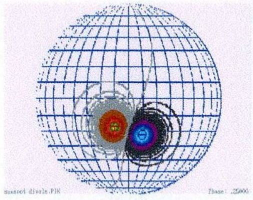
Parameters:	Radius-fraction	Longitude	Latitude	Charge
	$r_1=0.5$	$\varphi_1 = 90°$	$\delta_1 = 45°$	$Q_1=+1$
	$r_2=0.5$	$\varphi_2 = 270°$	$\delta_2 = -45°$	$Q_2=-1$

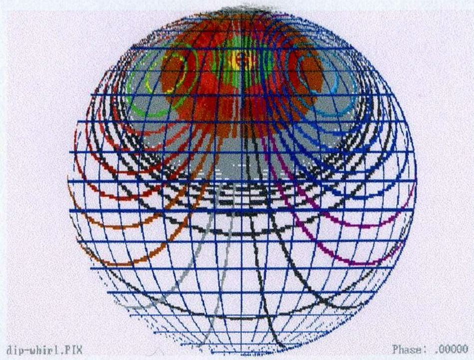
The sources with (virtual) magnetic charges can be located arbitrarily inside and outside the star.
The magnetic dipole is the elementary unit for the construction of magnetic potential fields.
The fields of numerous magnetic dipoles superpose linearly without any mutual interference.

Below: Construction of the global magnetic field of a solar-like starspot as an example of an extremely decentered magnetic dipole

Parameters:	Radius-fraction	Longitude	Latitude	Charge
	$r_1=0.88$	$\varphi_1 = 170°$	$\delta_1 = -27.5°$	$Q_1=+1$
	$r_2=0.92$	$\varphi_2 = 190°$	$\delta_2 = -32.5°$	$Q_2=-1$







Construction of the stellar magnetic field on sources and vortices

E. Gerth and Yu.V. Glagolevskij

The description of the magnetic field on the stellar sphere needs an appropriate analytical calculus, for which an expansion of spherical harmonics (Legendre polynoms) has been commonly adopted (see Oetken [5], Krause & Rädler [4], Bagnulo et al. [1]). However, although the mathematical treatment by Legendre polynoms yields an analytical function of the field distribution on the surface and its fitting to the observation, it conceals the physical meaning of the coefficients and the origin of the magnetic field.

We relate to a model of a star, in whose interior the magnetic field is generated by sources and vortices with arbitrary distribution in the space. The surface of a sphere like a star is penetrated by the lines of force from inside or from outside. Only the topographic surface structure of the magnetic field in the stellar atmosphere is observable and will be calculated for a model.

In the case of a stable star with a stationary field one has to account only for the magnetic sources, which act analogously to electric charges as virtual magnetic charges. The magnetic charge is the initial point of a magnetic potential, the gradient of which is the magnetic field strength. The field of a magnetic monopole is spherically symmetric, making a numerical treatment comfortable to develop an algorithm as a standard [2].

Since magnetic monopoles do not exist in reality, only magnetic dipoles are physically relevant. A dipole consists of two magnetic charges of opposite polarity. The magnetic moment of a such a dipole is a vector, being surrounded by a characteristic magnetic vector field. The fields of numerous elementary dipoles superpose linearly to multipoles and "super-multipoles". Also for the magnetic dipole a standard algorithm can be used. The superposition of two monopoles gives one more free parameter by the distance between the charge points.

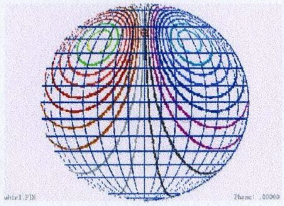
Similarly to the magnetic dipole the algorithm for the magnetic vortex is programmed by the differential operator curl, which is useful for the description of a magnetic field generated by an electric current streaming through a point inside the star. The real magnetic field of a star is a combination of different fields of dipoles and vortices which superpose linearly according to a definite theorem of the potential theory.

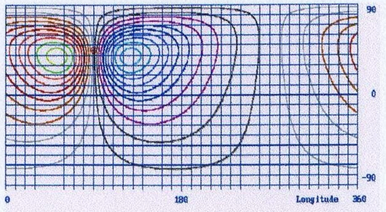
The magnetic vector field is defined by its coordinates in the interior of the star as well as in the whole surrounding space and can also be determined on any area you like. The most important area is the surface of a sphere – the face of a star, representing the cartographic map with all its topographic features of the magnetic field and the element distribution in the star's atmosphere, which will be integrated over the visible disk, convoluted by rotation, and transformed into phase curves of the integral magnetic field.

References

[1] Bagnulo S., Landi Degl'Innocenti, M., Landi Degl'Innocenti, E.: 1996, Astron. Astroph. **308**, 115
[2] Gerth, E.: 2001, Proc. Workshop on "Magnetic Fields across the Hertzsprung-Russell diagram", Santiago de Chile 15-19 Jan. 2001, eds. G. Mathys, S.K. Solanki, D.T. Wickramasinghe, **248**, 333
[3] Gerth, E.: 2002, Proc. IAU Symposium "Modelling of Stellar Atmospheres", eds. N.E. Piskunov, W.W. Weiss, D.F. Gray, in press
[4] Krause F., Rädler K.-H.: 1980, "Mean-Field Magnetohydrodynamics and Dynamo Theory", Akademie-Verl., Berlin, and Pergamon Press, Oxford
[5] Oetken L.: 1977, Astron. Nachr. **298**, 197; 1979, Astron. Nachr. **300**, 1

Magnetic Vortices





Structure of the field of an eccentric vortex on the surface of a sphere

The field of a vortex is determined by 6 parameters (3 local coordinates, 3 components electr. current):

Parameters:	Radius-fraction	Logitude	Latitude	$\mathbf{I}_x$	$\mathbf{I}_y$	$\mathbf{I}_z$
	$r=0.5$	$\varphi=90°$	$\delta=45°$	0	1	0

The vortex constitutes the closed magnetic lines of force around an axial vector, which is located by the 3 coordinates r, φ, δ and directed by the spatial motion of an electrical charge – the 3 vector components of the electrical current I related to the Cartesian coordinates x, y, z.

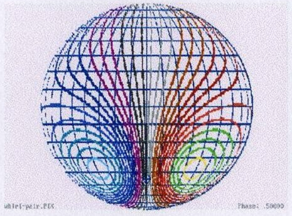
(The 3 coordinates x,y,z are related to the sphere by the radius r and can be written as spherical coordinates with 3 parameters: I current absolute, λ horizontal, ϑ azimuthal)

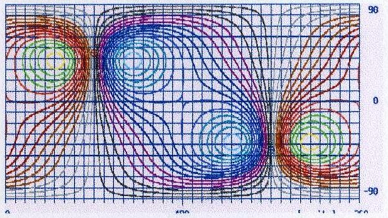
The magnetic field strength is derived by the vectorial differential operator curl I

$$\mathrm{curl}\, I = \mathbf{i}\,(\partial \mathbf{I}_z/\partial y - \partial \mathbf{I}_y/\partial z) + \mathbf{j}\,(\partial \mathbf{I}_x/\partial z - \partial \mathbf{I}_z/\partial x) + \mathbf{k}\,(\partial \mathbf{I}_y/\partial x - \partial \mathbf{I}_x/\partial y)\ ,$$

which is programmed like the gradient as a standard algorithm.

The fields of vector curls can be superposed linearly like and to the fields of potential gradients.





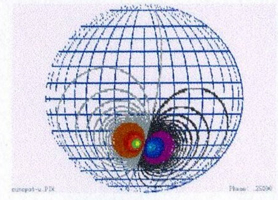
Globe and map of the magnetic field of two whirls, located diametrically at r = 0.5

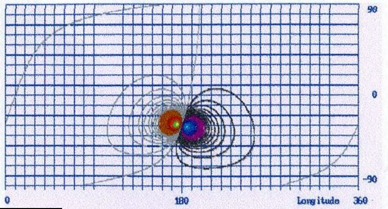
Parameters:	Radius-fraction	Longitude	Latitude	horizontal	azimuthal	Current
	$r_1=0.5$	$\varphi_1 = 90°$	$\delta_1 = 45°$	1	0	$I_1=+1$
	$r_2=0.5$	$\varphi_2 = 270°$	$\delta_2 = -45°$	1	0	$I_2=-1$

The whirls are positioned at the same points like the sources on the left side. The curls correspond by direction to the polarity of the charges but are directed horizontally ($\lambda = 1$, $\vartheta = 0$).
The closed lines of force, which penetrate two times the surface of the star, make two poles with opposite polarity – in contrast to sources with virtual magnetic charges.
All magnetic field structures might be modelled by arrangements of sources or/and whirls.

Below: Representation of a solar-like starspot by means of a whirl

Parameters:	Radius-fraction	Longitude	Latitude	horizontal	azimuthal	Current
	$r=0.9$	$\varphi = -90°$	$\delta = -30°$	1	0.25	$I=+1$





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