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Construction of a Stellar Magnetic Field on Sources and Vortices

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Abstract. A magnetic field is a vector field which fills space and penetrates any body with its lines of force. The stellar magnetic field is modelled on the basis of magnetic sources and vortices and constructed for the observable spherical surface of a star. The topographical structure of the magnetic field on the star's surface is represented as a map with topographical features as, e.g., poles and iso-magnetic lines and areas[1].

1. Introduction

The description of a magnetic field on a stellar sphere requires an appropriate analytical framework, for which an expansion in terms of spherical harmonics (Legendre polynomials) has been commonly adopted (see Oetken (1977), Krause & Rädler (1980), Bagnulo et al. (1996)). Although the mathematical treatment using Legendre polynomials yields an analytical function for the surface field and a fit to observations, it conceals the physical meaning of the coefficients and the origin of the magnetic field.

We consider a model of a star, whose magnetic field is generated by *sources* and *vortices*, which are arbitrarily distributed in space. The magnetic vector field is defined by its coordinates in the interior of the star as well as on any surface surrounding the star. The most important surface is a sphere – the observable part of a star – which may be represented as a cartographic map with all the topographic features of the magnetic field and the element distribution in the star's atmosphere. The radiation from surface elements, which contain information on the magnetic field, will be integrated over the visible disk, convolved by rotation, and transformed into phase curves of the *integral magnetic field*. The calculation of the spatial vector field and the mapping of the stellar surface field is performed by a computer program, which contains the standard algorithms for the *source* (gradient) and the *vortex* (curl). The concept of a "magnetic charge distribution" is outlined and applied by Gerth et al. (1997, 1998). Use of this method have made also Khalak et al. (2001, 2002).

[1]Poster representation: www.ewald-gerth.de/105pos.pdf

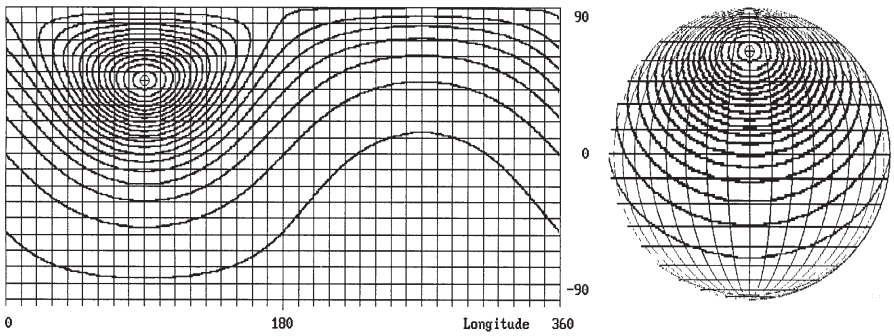


Figure 1. Map and globe of the field structure of an eccentric monopole on the surface of a sphere. The field of a monopole is determined by four parameters (three local coordinates x, y, z and charge Q). A monopole of unit charge is located at fractional radius $r = 0.5$, longitude $\varphi = 90°$ and latitude $\delta = 45°$.

2. Magnetic sources

In the case of a stable star with stationary field, one has to account only for the *magnetic sources*, which act analogously to electric charges as *virtual magnetic charges*. A magnetically charged *monopole* is the starting point of a *magnetic potential*, the gradient of which is the *magnetic field strength* of a point-like source. The field of a magnetic monopole is spherically symmetric.

A monopole field source with charge Q is surrounded by a spherical potential, $U = Q/(2\pi r)$ with r as the radial coordinate. The field strength is derived from the potential by the gradient of the potential:

$$\nabla U = \mathbf{i}\frac{\partial U}{\partial x} + \mathbf{j}\frac{\partial U}{\partial y} + \mathbf{k}\frac{\partial U}{\partial z}$$

where $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$ are Cartesian unit vectors. For the numerical calculation of a spherical potential field, we use the algorithm developed by Gerth & Glagolevskij (2001), which serves as the fundamental standard for the superposition of monopole fields. In this way a complex structure can be represented as a superposition of elementary structures. This superposition is possible because of the linearity of the differential operator, ∇, which is calculated simply as the sum of the vector components of the different fields.

The monopole field is a physical reality for electrical monopoles. However, since magnetic monopoles do not exist, only *magnetic dipoles* are physically relevant. The magnetic moment of a magnetic dipole is a vector, comprising a characteristic magnetic vector field. A magnetic dipole consists of two magnetic charges of opposite polarity. It is therefore characterised by 8 parameters. The superposition of two equally charged monopoles to make a dipole reduces the number of parameters to 7.

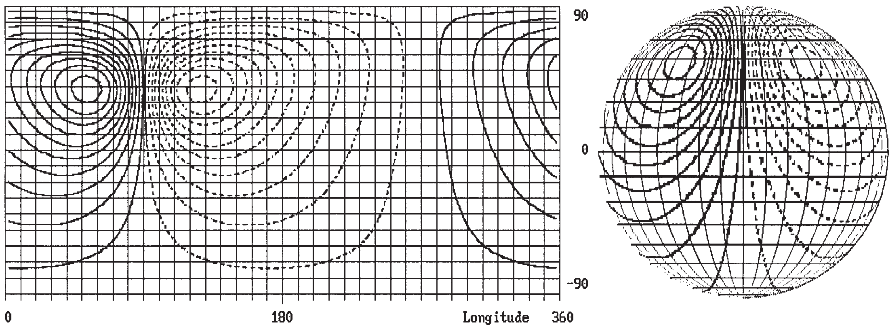


Figure 2. Map and globe of the field of an eccentric vortex on the surface of a sphere. Solid lines: positive region; dotted lines: negative region. The field of a vortex is determined by six parameters (3 local, 3 electric). The fractional radius is $r = 0.5$, the longitude $\varphi = 90°$, the latitude is $\delta = 45°$ and $I_x = I_z = 0$, $I_y = 1$.

The algorithm, in principal only valid for a monopole, can be used also for oppositely-charged monopoles which combine to form dipoles, quadrupoles, multipoles and "super-multipoles" (Gerth & Glagolevskij 2003).

3. Magnetic vortices

A real magnetic field is a combination of different fields of dipoles and vortices which superpose linearly, according to a theorem in potential theory. Like the gradient for the magnetic dipole, the algorithm for the magnetic vortex is based on the linear differential operator **curl**. This is useful for describing a magnetic field generated by an electric current streaming through a point inside the star.

A vortex constitutes the closed magnetic lines of force around an axial vector with origin at spherical coordinates r, φ, δ and direction determined by the spatial motion of an electrical charge through Cartesian space. The three vector components of the electrical current, I, with origin at Cartesian coordinates x, y, z on the sphere with radius r, can be written in spherical coordinates also with three parameters: the magnitude of the current, I, and λ, the horizontal component and ϑ, the azimuthal component.

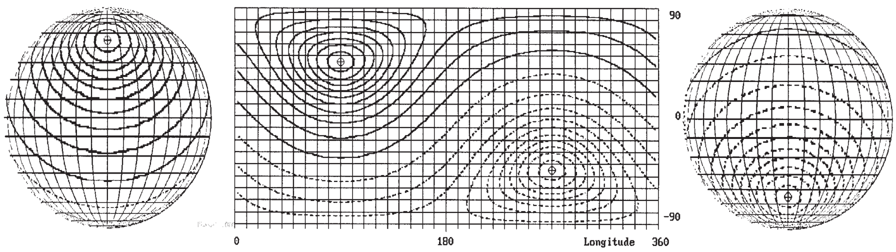
The field strength of a vortex is derived by the vectorial differential operator

$$\nabla \times \boldsymbol{I} \;=\; \mathbf{i}\left(\frac{\partial I_z}{\partial y} - \frac{\partial I_y}{\partial z}\right) + \mathbf{j}\left(\frac{\partial I_x}{\partial z} - \frac{\partial I_z}{\partial x}\right) + \mathbf{k}\left(\frac{\partial I_y}{\partial x} - \frac{\partial I_x}{\partial y}\right)$$

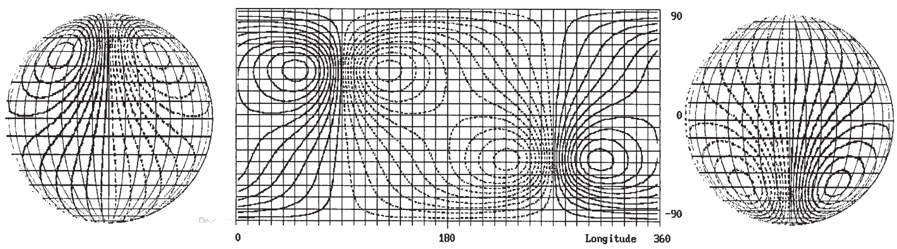
which is programmed like the gradient as a standard algorithm.

4. Comparison of the surface field structures of sources and vortices

Sources and vortices produce characteristic field structures on the surface of the sphere, which we compare using the same coordinates. Fig. 1 shows a map of



Radius-fraction	Longitude	Latitude	Charge
$r_1 = 0.5$	$\varphi_1 = 90^o$	$\delta_1 = +45^o$	$Q_1 = +1$
$r_2 = 0.5$	$\varphi_2 = 270^o$	$\delta_2 = -45^o$	$Q_2 = -1$



Radius-fraction	Longitude	Latitude	horizontal	azimutal	Current
$r_1 = 0.5$	$\varphi_1 = 90^o$	$\delta_1 = +45^o$	1	0	$I_1 = +1$
$r_2 = 0.5$	$\varphi_2 = 270^o$	$\delta_2 = -45^o$	1	0	$I_2 = -1$

Figure 3. Maps and globes of dipoles and vortices constructed on the same coordinates.
Top: central magnetic dipole with separated magnetic charges.
Bottom: field of two vortices, located diametrically at $r = 0.5$.
(Positive regions: solid lines; negative regions: dotted lines)

a positive monopole located at half of the radius with an emerging unipolar magnetic pole on the surface. The vortex with the same coordinates and the axial vector directed horizontally (parallel to the equator) produces two opposite poles on the surface as shown in Fig. 2.

An objection might be raised against a "magnetic monopole" because it is not a physical entity. Therefore, we compare a dipole consisting of two oppositely charged sources with a system of two vortices. Fig. 3 demonstrates the effect of equally arranged sources and vortices on the surface field structure. The curl directions show the polarity of the charges but are directed horizontally ($\lambda = 1$, $\vartheta = 0$). The closed lines of force, which twice penetrate the surface of the star, generate two poles with opposite polarity on the stellar surface. This is in contrast to magnetic sources with virtual charges.

The number of spatial arrangements of numerous sources and vortices is, of course, infinite. The modelling of magnetic field structures is traced back to the origin of geometrically defined points, which we recognize as the *eigenvalues* of the magneto-hydrodynamic system – by analogy to the *eigenfrequencies* of an oscillating system.

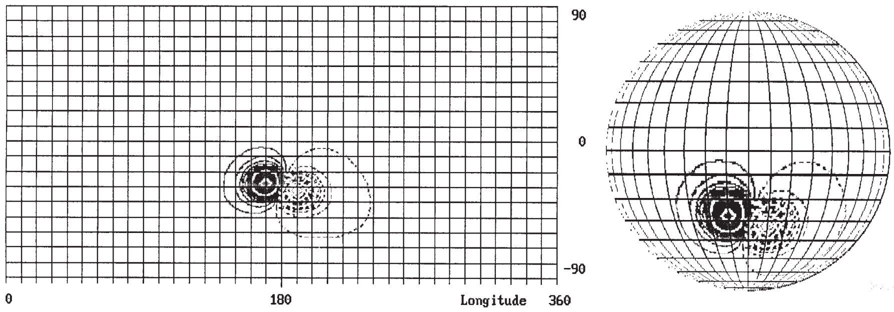


Figure 4. Construction of the global magnetic field of a solar-like starspot as an example of an extremely decentered magnetic dipole.

8 parameters of a pair of magnetic sources:

Radius-fraction	Longitude	Latitude	Charge
$r_1 = 0.88$	$\varphi_1 = 170^o$	$\delta_1 = -\,27.5^o$	$Q_1 = +1$
$r_2 = 0.92$	$\varphi_2 = 190^o$	$\delta_2 = -\,32.5^o$	$Q_2 = -\,1$

6 parameters of a magnetic vortex:

Radius-fraction	Longitude	Latitude	horizontal	azimutal	Current
$r = 0.9$	$\varphi = -\,90^o$	$\delta = -\,30^o$	1	0.25	$I = +1$

5. Mapping of vector fields on the star's surface

The mapping of surface structure of the stellar magnetic field can be achieved by modelling on the basis of a *magnetic charge distribution* (Gerth et al. 1997, 1998). The magnetic dipole field is the elementary unit for the construction of magnetic potential fields, which superpose without any mutual interference.

The sources with (virtual) magnetic charges can be located arbitrarily inside and outside the star; this means that the sources need no connection to the geometrical center of the star. As an example of an extremely decentered dipole we demonstrate in Fig. 4 the model of the global magnetic field of a solar-like star spot (Gerth & Glagolevskij 2003). The dipole is supposed to be located just beneath the surface. Positive and negative poles of the spots lie close together. The microstructure of a real sunspot might be modelled by a multiplicity of elementary dipoles and elementary vortices. The construction of the fields of solar-like spots with a vortex gives nearly the same picture but corresponds even better to solar physics, because the bundled magnetic lines of force are closed from the interior to the corona and penetrate the surface twice as loops.

The inner regions of the star and the surface fields can be constructed as well on sources as on vortices. In the outer regions, however, there is a fundamental difference between the fields of sources and vortices on physical grounds. The cause for the potential field is a charged source, but for the closed magnetic lines of force this is an electrical current, which can flow only in the interior of the stellar body. In contrast to this, charges might be located also on external points, e.g., on a companion of a binary system. Whether or not such a configuration might exist in nature can be proved only by observation.

6. Conclusion

The analogy between hydrodynamics, where the vectorial velocity field is completely determined by its "wells and whirls", and magneto-hydrodynamics leads to the idea of constructing the magnetic vector field by "sources and vortices". The modelling of stellar magnetic fields calls for an appropriate framework, which is suited for the programming of standard algorithms of the fields of a source and a vortex. These algorithms are those of the differential operators **grad** and **curl** with the corresponding coordinate transformations, suitably realized in a computer program. The program is used as a tool for the analysis of the magnetic field structure of observed magnetic stars.

The range of applicability of this method is much wider than the commonly used method using spherical harmonics, which is limited to the surface of a sphere. The coefficients of the spherical harmonics are derived, and therefore secondary quantities, the physical meaning of which is not clear. Nevertheless, they could be taken as solutions of Legendre's differential equation, which leads by an inverse procedure to the intrinsic eigenfunctions.

The method presented here does not use spherical harmonics. The theory and its application in a computer program starts from the origin of the field with its "eigenvalues", the *sources* and *vortices*, which describe the vectorial field in space. This includes any area, such as the surface of a sphere.

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