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# A quadrupole model of the magnetic field of $\beta$CrB

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**Abstract.** A model of the magnetic field of the star $\beta$CrB is constructed by the method of the "Magnetic Charge Distribution". The magnetic field is described by superposition of two decentered dipoles, which are arranged in the equatorial plane perpendicularly to the axis and oppositely directed to the center of the star. The dipoles form an irregular quadrupole and produce on the surface four magnetic spots with a maximal magnetic field strength at the poles of $B_p = 14.5$ kG. The angle of declination of the rotation axis to the line of sight is $i = 8^o$ to $15^o$. In consideration of all aspects a strong influence of an inhomogeneous distribution of chemical elements onto the measurable phase curves of the effective magnetic field has to be assumed. The star apparently has passed a convective phase in an early stage of evolution[1].

The star $\beta$CrB is one of the most investigated magnetic stars because of its brightness and sharpness of the spectral lines. An early attempt to explain the phase relation of the effective field $B_e$ by an *equally symmetric rotator model* using an expansion of Legendre functions goes back to Oetken (1977). She derived from the observed phase curve a prevailing quadrupole moment. Modelling of the magnetic field of $\beta$CrB has recently been done also by Bagnulo et al. (2000), who describe the field by a $second-order\ expansion$ on the base of a centered dipole plus a non-linear quadrupole.

We analyze the magnetic field structure of chemically peculiar stars by the modelling method of the "*Magnetic Charge Distribution*" (MCD), which is described by Gerth et al. (1997, 1998, 2000) and Gerth & Glagolevskij (2000, 2001) and applied to some real stellar objects by Gerth et al. (1997, 1998, 1999) and Glagolevskij & Gerth (2000, 2001, 2002). By this method we try to reproduce the modelling also for $\beta$CrB to get more insight into the origin of the magnetic field and its structure on the star's surface and – as far as the reduction of the observable surface field to its virtual sources is valid – also into its interior.

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From all available observational data we chose the phase relations $B_s(P)$ from WOLFF & WOLFF (1970) and MATHYS et al. (1997) (Fig. 1a). The first data are taken from old photographic observations by the lines of Cr, Ti, and Fe. The second data were obtained by the lines of Cr and Fe with a CCD-camera. Further we chose the phase curves $B_e(P)$ with magnetic field measurements of the hydrogen line H$\beta$ by BORRA & LANDSTREET (1980) (Fig. 1b). In contrast to the metals hydrogen is homogeneously distributed over the star's surface. We know by experience (see GLAGOLEVSKIJ & GERTH, 2002), that the phase curves obtained by metallic lines are distorted to some degree because of the inhomogeneous distribution of metal atoms over the surface. The observed phase curves are arranged using the ephemeris after KURTZ (1989)

$$\mathrm{JD}_{magnetic\ max} = 2434204.70 + 18.4868\ \mathrm{E}.$$

The angle $i = 15^o$ found by ours corresponds best of all to both phase relations, that we have determined by means of the least squares optimization. The rather small magnitude of $B_e < 1$ kG together with the large quantity $B_s \approx 5.5$ kG argues for the assumption, that the magnetic poles are located at the limb of the visible disk, e.g., close to the equatorial plane, having in mind that the star is visible almost from its pole.

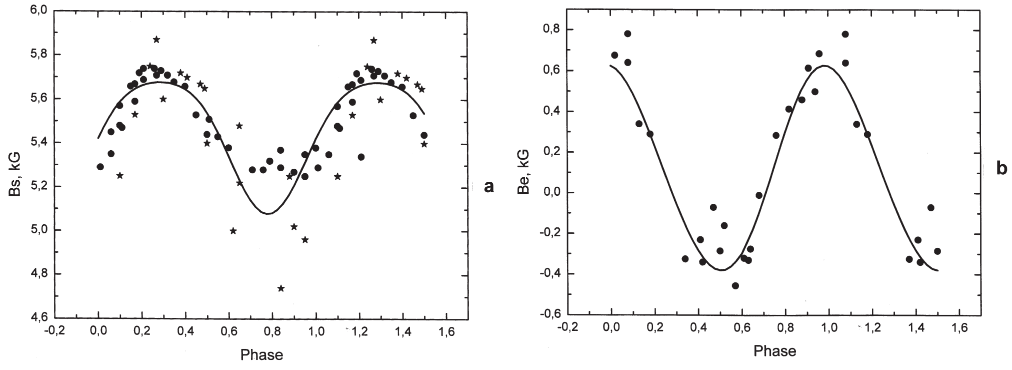

Figure 1. Observed phase relations for $\beta$CrB.
Solid lines – calculated phase curves; dots – observations
a) averaged magnetic field $B_s$, b) effective magnetic field $B_e$
dots – data from MATHYS (1997), data from BORRA & LANDSTREET (1980)
asterisks – data from WOLFF & WOLFF (1970)

The magnitude of the magnetic field equals 14.4 kG on the magnetic poles. The distance of the charges from the center is $r$=0.3 of the stellar radius. All charges have the same relative value 525, which is fitted to the observed magnetic field strength on the surface. The dipoles are arranged in the equatorial plane.

A characteristic feature of the phase relations is, that the maximum of the curve of $B_s(P)$ passes by at the moment, when $B_e \approx 0$. The modelling of some other stars (HD 126515, GLAGOLEVSKIJ & GERTH, 2000) has proven, that such a situation is possible in the case of a non-central (decentered) arrangement of a dipole. For the compensation of the deep minimum of $B_s(P)$ near to the phase 0.8 it is inevitable to introduce still a further dipole, which has to be arranged on the opposite side to the first one.

We found, that the observed phase curves can be explained by assuming two by polarity oppositely directed dipoles, whose magnetic charges are located at the distance of 0.3 stellar radius from the center of the star. We call the combination of two arbitrarily arranged dipoles an *irregular quadrupole*.

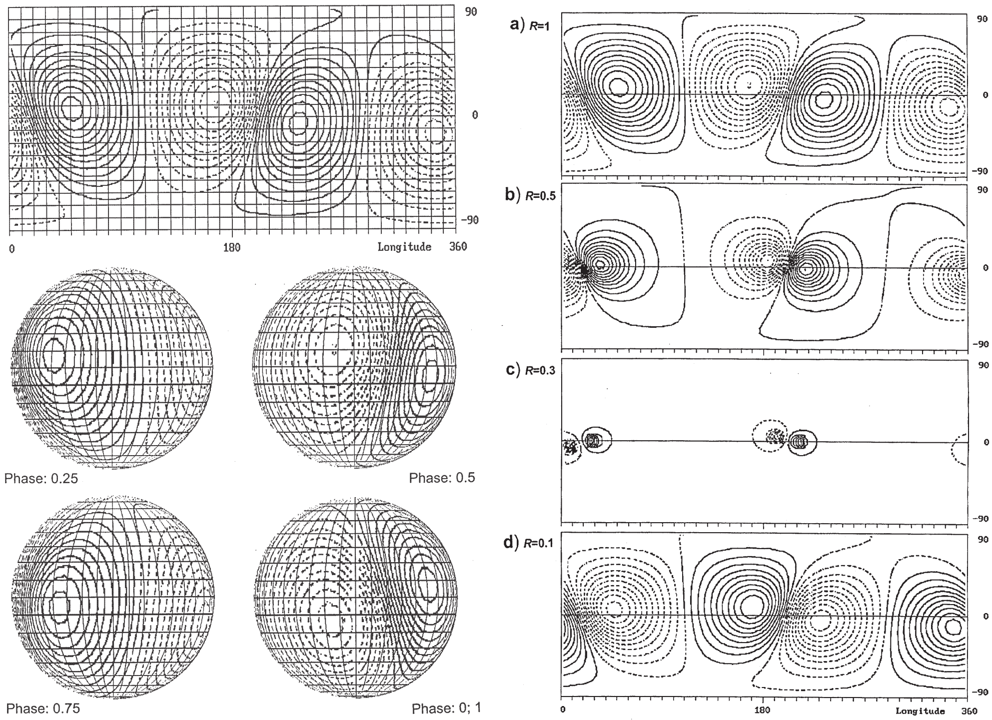

Figure 2. Distribution of the magnetic field strength of $\beta$CrB
Solid lines – positive regions; dashed lines – negative regions
(Computer program written by E. Gerth)
a) over the surface at the radius $R$=1 b) inside the star on different levels of R
Above: Pseudo-Mercator map with 4 magnetic extrema of alternative polarity
Beneath: Globes for four rotation phases
Parameters:
$R$=1
$R$=0.5
$R$=0.3
$R$=0.1

| No | $Q$ | $r$ | $\lambda$ | $\delta$ |
|---|---|---|---|---|
| 1 | – 525 | 0.3 | 351 | -6.5 |
| 2 | +525 | 0.3 | 12 | 0 |
| 3 | – 525 | 0.3 | 176 | 5.3 |
| 4 | +525 | 0.3 | 196 | 0 |

Our model does not predict a significant gradient of the field as a function of the depth. If we assume a thickness of the atmosphere of $10^3$ km, then the maximal difference of the magnetic surface field on the extreme levels amounts to 25 G. This has been calculated by means of the program used by the authors, inserting the radii on the surface and the radius 1000 km higher. The change of the field structure with the depth starting from the surface, goes at first rather slowly, as we can see in Fig. 2b. This slow course changes strikingly at 0.3 of the stellar radius, where the sources are located at points of singularity. In the infinitesimal vicinity of the singularities the field strength rises up to infinity. Closer to the center the field strength changes the polarity but has a similar structure on an inner sphere at 0.1 $r$.

This is correct for real point-like sources, as it is realized for electrical charges. However, the analogy between electrical and magnetic charges has its limitations. The closed magnetic lines of force have no singularity at all. Therefore, the inner structure of the magnetic field is not simply founded on a few magnetic sources. Nevertheless, the *magnetic charges* as *virtual sources* render an advantageous base for programming and fits the better to the reality the more the distance from the singularities increases.

Our result confirms the assertion expressed in a paper of BAGNULO et al. (2000) that it is impossible to describe even opproximately the structure of the magnetic field of $\beta$CrB only by means of a traditional dipole model (LANDSTREET, 1980).

In present time the hypothesis of a relict origin of the stellar magnetism in CP stars is commonly supported. However, in this case the structure of a field similar to that of $\beta$CrB is hardly possible. Such a structure might easier be explained by the hypothesis of a *magnetic dynamo.* If we locate this star on evolutionary tracks, then we can see that its way passes the early stages of the evolution through regions occupied by TTau stars; that means, the star could pass in its early stages a phase of convective instability, when a magnetic field could be generated. In this case we do not observe at $\beta$CrB the original relict field, but a secondary one, generated by the effect of a stellar dynamo. $\beta$CrB belongs to a group of SrCrEu type stars, which went in an early stage of evolution through a convective phase.

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