Modelling of Stellar Atmospheres
IAU Symposium, Vol. 210, 2003
N. E. Piskunov, W. W. Weiss, D. F. Gray, eds.

Modelling of stellar magnetic fields by superposition of elementary dipoles

Ewald Gerth
D-14471 Potsdam, Gontardstr 130, Germany.
e-mail: ewald-gerth@t-online.de

Yurij V. Glagolevskij [1,2]

[1] *Special Astrophysical Observatory of Russian AS, Nizhnij Arkhyz 357147, Russia. e-mail:* glagol@sao.ru
[2] *Isaac Newton Institute of Chile, SAO Branch*

Abstract. Modelling of magnetic fields on the stellar surface is performed by calculation of the spherical fields of monopoles, which combine to dipoles and multipoles. Elementary magnetic dipoles constitute the components of any magnetic field structure in stars by superposition[1].

Stellar magnetic fields can only be observed from the surface of the star in integral light, including a variety of convolution processes. The deconvolution, however, proves to be an ill-posed inversion for lack of information, which would be overcome by straightforward modelling and fitting to the observation.

We relate to a model of a star, in whose interior the magnetic field is generated by sources and whirls. In the case of a stable star with a stationary field one has to account only for the magnetic sources, which act analogously to electric charges as *virtual magnetic charges*. The calculation is based on an algorithm for the spherical monopole field (Gerth et al. 1997, 1998, 2001).
An elementary "*magnetic monopole*" is determined by 4 parameters.

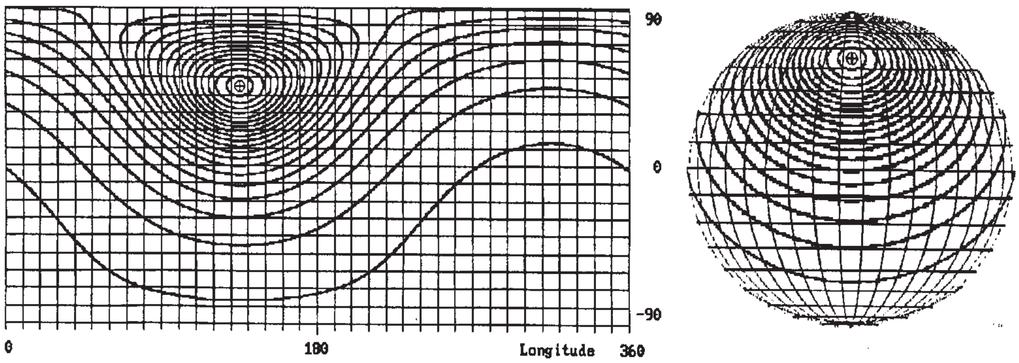

Figure 1. Mercator-map and spherical projection (globe) of a positively charged monopole ⊕. The *iso − magnetic* lines mark the topographic location of equal magnetic field strength.

1. Charge	$Q = 1$	(relative units)	3. Longitude	$\varphi = 135^o$
2. Radius	$r = 0.5$	(fraction of the radius)	4. Latitude	$\delta = 45^o$

[1] IAU Symposium 210 in Uppsala/Sweden, 2002, June 17-21. Poster: **www.ewald-gerth.de/102pos.pdf**

Since magnetic monopoles do not exist in reality, only magnetic dipoles are physically relevant. The magnetic moment of a such a dipole is a vector, being surrounded by a characteristic magnetic vector field. The fields of numerous elementary dipoles superpose linearly to multipoles and “super-multipoles” – as the molecular dipoles in ferromagnetic bodies. This is demonstrated, e.g., by two magnetic dipoles at the same position, the fields of which build again a dipole field with the vectorial addition of the magnetic moments.

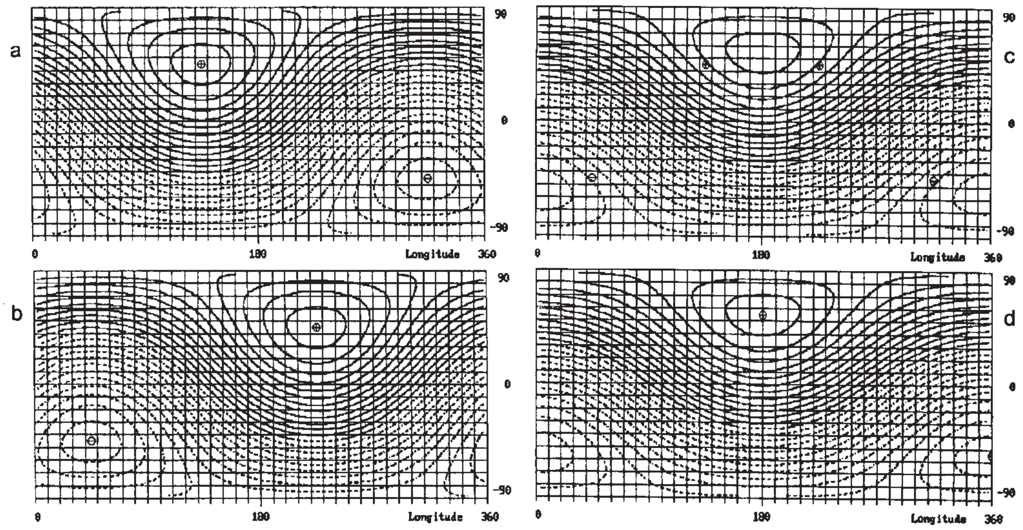

Figure 2. Superposition of 2 central magnetic dipoles with two pairs of charged sources. Solid iso-magnetic lines - region of positive magnetic field strength; dashed lines - negative field

a) $Q = +1$ $r = 0.001$ $\varphi = 135^o$ $\delta = +45^o$
$Q = -1$ $r = 0.001$ $\varphi = 315^o$ $\delta = -45^o$

b) $Q = -1$ $r = 0.001$ $\varphi = 45^o$ $\delta = -45^o$
$Q = +1$ $r = 0.001$ $\varphi = 225^o$ $\delta = +45^o$

c) The superposition yields a resultant dipole with the 4 φ, δ-coordinates marked by $\oplus$ $\ominus$

d) $Q = +1$ $r = 0.001$ $\varphi = 180^o$ $\delta = +55^o$
$Q = -1$ $r = 0.001$ $\varphi = 0^o$ $\delta = -55^o$

The magnetic field is defined by its coordinates in the interior of the star as well as in the whole surrounding space and can also be determined on any area you like. The most important area is the surface of a sphere – the face of a star, representing the cartographic map with all its topographic features concerning the magnetic field and the element distribution in the star’s atmosphere, which will be integrated over the visible disk, convoluted by rotation, and transformed into phase curves of the integral magnetic field.

The magnetic charges can be arranged anywhere in the interior of the star. A central dipole has a symmetrical arrangement of the charges in regard to the center with a determined magnetic moment. The magnetic moment $M = Q{*}l$ is an intrinsic constant of the magnetic dipole, which means that the charge Q and the length l are related to each other reciprocally. The vector addition of the magnetic moments of two dipoles yields a similar resultant dipole field only in the case of an infinitesimal distance $l \rightarrow 0$ and $Q \rightarrow \infty$. We consider for the “Magnetic Charge Method” dipoles with a definite distance between the oppositely charged field sources. With increasing distance of the sources the sites of the poles are shifted away from the φ-δ-coordinates of the sources, which is particularly evident for a dipole in the equatorial plane (Fig. 3a).

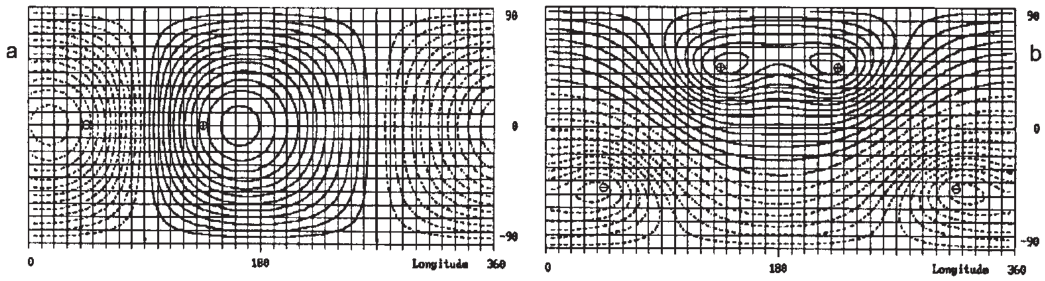

Figure 3. The fields of magnetic dipoles with essential distance of the sources from the center
a) $Q = +1$ $r = 0.1$ $\varphi = 135^o$ $\delta = 0^o$
$Q = -1$ $r = 0.1$ $\varphi = 45^o$ $\delta = 0^o$
b) The field of the combination of two dipoles corresponding to Fig. 2c with $r = 0.5$

If the magnetic dipole is displaced from the center, then we call it a "*decentered dipole*". Multipoles consist of a definite number of regularly decentered dipoles.

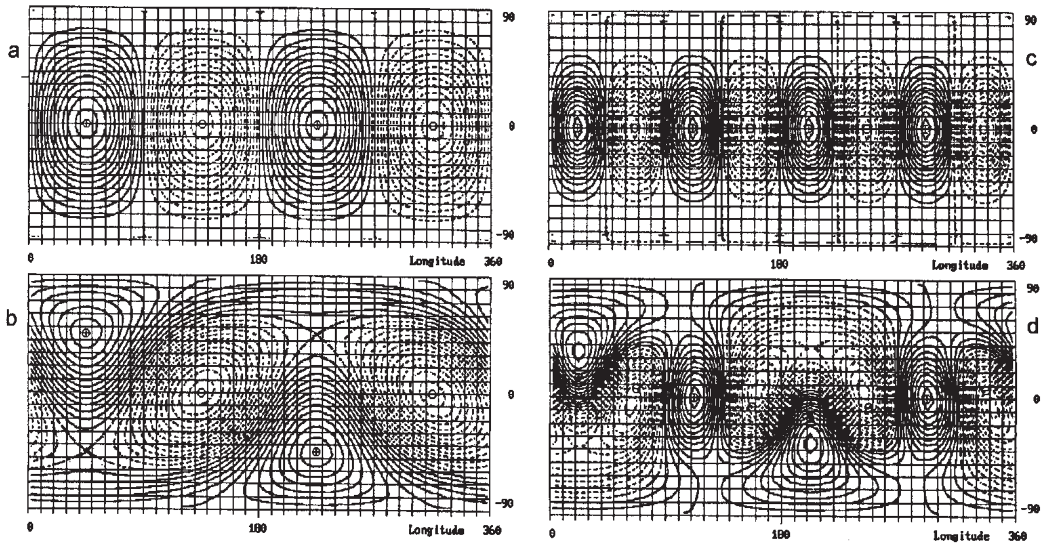

Figure 4. Multipoles as arrangements of decentered dipoles around the center of the star:
Magnetic quadrupole – two decentered dipoles with antiparallel magnetic moments perpendicularly to the radius vector
a) in the equatorial plane
b) in a plane tilted to the equator by 45^o
Magnetic octupole – four decentered dipoles around the center in a ring with magnetic moments antiparallel and perpendicular
c) like a)
d) tilted by 15^o

Correspondingly to the *magnetic charges* the *elementary magnetic dipoles* may be arranged arbitrarily within the stellar body by position and by direction. The combination of elementary dipoles enables one to model different magnetic bodies: rod, cubic, cylinder, ellipsoid etc. Especially interesting is the field of an area of a circle, set with elementary dipoles and forming a "magnetic sheet". The same magnetic field is produced by a circularly streaming electric current as a whirl, such as we can assume circling in the star both in cases of a stellar dynamo and of a frozen-in relict magnetism.

As an example for a magnetic "super-multipole" we demonstrate the field structure of a magnetic sheet, which lies as a circular disk in the x, y-plane of the star with $0.5R$ in the center and tilted by 30^o to the z-axis. With an arrangement of dipoles after Fig. 5 we can construct a "super-multipole":

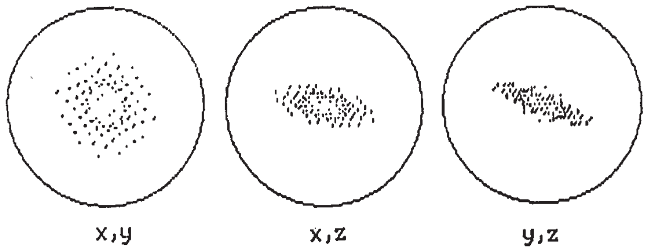

Figure 5. "Super-multipole" of 80 dipoles as double layer of positive and negative monopoles. The grating looks pillow-like and empty in the middle because of the shifting of the charges. All dipoles have equally two oppositely charged field sources. Each dipole has a distance of the 2 point-like charges of $0.01R$ (160 points). The dipoles are set within the circle $r = 0.5R$ in a grating of 10*10. The circle in the x, y-plane is tilted to the equator by 30^o.

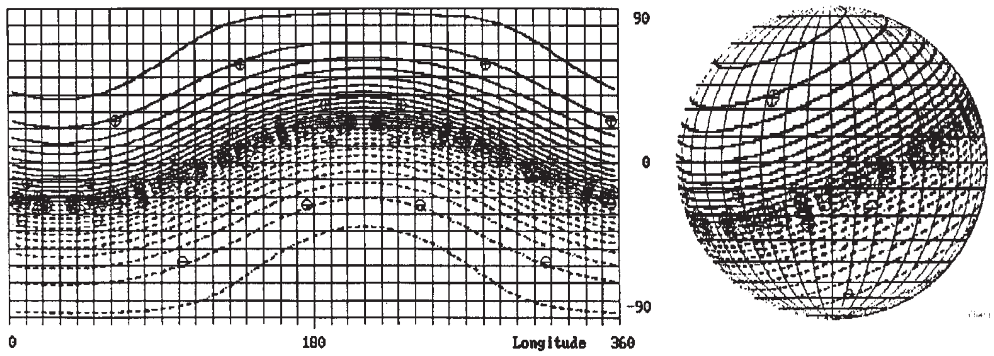

Figure 6. Mercator-map and globe of the circular magnetic sheet approximated by 80 dipoles, represented with the cartographic coordinates of the sources $\oplus$ $\ominus$ and iso-magnetic lines.

The resultant field is similar to that one of a dipole with the coordinates φ=210^o and δ=70^o. The replacement of the super-multipole by a simple dipole, however, shows that the gradient of the magnetic field strength at the equator is steeper in the case of the magnetic sheet, because the double-layer is there more extended to the surface. By this way the external structure of the field allows to conclude on the interior structure of the star.

In principle, all magnetic structures can be modelled by combination of elementary magnetic dipoles. The philosophy of the MCD-method relates to a definite theorem of the potential theory, according to which any field configuration is produced by superposition of the fields of numerous point-like sources.

References

Gerth E., Glagolevskij Yu.V., Scholz G., in: Stellar magnetic fields, Eds.: Glagolevskij Yu.V., Romanyuk I.I., Moskow, 1997, p. 67

Gerth E., Glagolevskij Yu.V., Scholz G., 1998, Contrib. Astron. Obs. Skalnaté Pleso, 27, 455

Gerth E., Glagolevskij Yu,V., 2001, in: Magnetic fields across the Hertzsprung-Russell diagram, Eds.: Mathys, G., Solanki S.K., Wickramasinghe, D.T., 248, Santiago de Chile, p. 333